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Modeling of bubble surface area flux in an industrial rougher column using artificial neural network and statistical techniques

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ABSTRACT

Previous studies in mechanical and column flotation cells have shown that *bubble surface area flux (S_b)* is an appropriate indicator of gas dispersion in a flotation cell which has a relatively strong correlation with flotation rate constant. In the present investigation, based on extensive tests conducted in an industrial Metso Minerals CISA flotation column (4 m in diameter and 12 m in height) in a rougher circuit, S_b as a function of the most significant operating variables which affect gas dispersion in a flotation column (i.e. *superficial gas velocity*, *slurry density (solids%)* and *frother dosage/type*) was modeled using artificial neural network (ANN) and statistical (non-linear regression) techniques. The models were developed taking into consideration a data set consisting of 82 experimental tests conducted in an industrial rougher column (at a copper concentrator in Iran) operating under a variety of experimental conditions.

This paper outlines the development of the models and validation using a number of randomly selected datasets. Limitations of the present models are discussed and comments and recommendations on further investigations are given.

1. Introduction

Previous studies have shown that bubble surface area flux (S_b) is a good measure of the gas dispersion in a flotation cell (Gorain et al., 1997; Massinaei et al., 2009). The S_b is a significant feature of the gas dispersed phase in a flotation cell which combines the effect of mean bubble size (d_b) and *superficial gas velocity* (J_g) as (Finch and Dobby, 1990):

$$S_b = \frac{6J_g}{d_b} \qquad (1)$$

where S_b = bubble surface area flux (cm^2/cm^2 s); J_g = superficial gas velocity (cm/s); d_b = mean bubble size (cm).

Previous findings have also demonstrated a good correlation between flotation rate constants (i.e. collection and overall rate constants, k_c and k) and S_b as a result of a number of experiments performed in industrial mechanical and column flotation cells (Gorain et al., 1997, 1998; Massinaei et al., 2009). These results indicated that S_b could be a significant parameter for control and optimization of flotation circuits. Thus, modeling of S_b as a function of the most significant operating variables which affect gas dispersion properties in a flotation cell would be useful (Gorain et al., 1999).

This paper discusses the development of empirical and artificial neural network models which enable to predict S_b in an industrial rougher column treating copper ores and validation of the models.

2. Experimental details

Industrial experiments were carried out in a flotation column in rougher circuit of Miduk copper concentrator, located in the south-east of Iran. In this plant, around 18000 tpd crushed ore with copper grade of around 1% is processed to produce a concentrate with a copper content of over 30%. Miduk is one of the unique copper processing plants which operate with columns in rougher circuit. The rougher circuit is consisted of five parallel Metso Minerals CISA flotation columns 12 m in height and 4 m in diameter, each 150 m^3.

As mentioned before, the aim was to develop models to predict S_b in terms of some important operating variables which could be measured easily in the plant. For this purpose, the most significant variables which could affect gas dispersion in a flotation column including *superficial gas velocity*, *slurry density (solids%)*, *frother dosage* and *frother type* were chosen. The operating conditions and the range of variables utilized in industrial experiments are listed in Table 1.

A full factorial experimental design comprising four variables (three variables at three levels and one variable at two levels) was performed (i.e. 54 runs). Experiments were carried out

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Table 1
Operating conditions of industrial experiments. Superficial wash water velocity = 0.04 cm/s; pH = 11.2–11.5; collector (X231) = 24 g/t; P_{80} = 90 μm.

Input variable	Range	Output variable
Superficial gas velocity, $J_g = Q_g/A_c$ (cm/s)	1.1, 1.6, 2	S_b (s^{-1})
Slurry density (solids%), ρ_{sl} (%)	24, 28, 32	
Frother dosage, C_f (ppm)	7, 13, 20	
Frother type, F_t	DF-200, DF-250/pine oil	

randomly based on design pattern. Afterwards, a number of repeated experiments and tests at more gas velocities (J_g = 1.3 and 1.8 cm/s) were conducted. In total, the results obtained from 82 industrial experiments were used for models development.

The S_b in various experiments was calculated from Eq. (1). *Superficial gas velocity* (J_g) was expressed as the ratio between the volumetric gas flow rate (Q_g) and the cross-sectional area of the column (A_c), i.e. $J_g = Q_g/A_c$. In the absence of bubble size measurement sensors, mean bubble size (d_b) was estimated using drift flux analysis. Drift flux technique, which was first introduced by Wallis (1962), has been used to estimate mean bubble size in flotation columns in a number of studies (e.g. Dobby et al., 1988; Yianatos et al., 1988; Banisi and Finch, 1994; Li et al., 2004). Gas hold-up (ε_g), superficial slurry velocity (J_{sl}) and slurry density (ρ_{sl}) were required for bubble size estimation procedure. Gas hold-up was measured using pressure difference technique. For this reason, two pressure taps were installed in collection zone of the column. Slurry density was measured using a Marcy scale and tailing flow rate was monitored by an ultrasonic flowmeter mounted on the discharge pipe of the column. After setting the operating variables and stability of the process, pressure values (from pressure gauges) and tailing flow rate (from tailing flowmeter) were recorded and slurry density was measured. Data were collected at 15 min intervals for 2 h in each run. More details of the process and experimental procedure have already been reported elsewhere (Massinaei et al., 2009).

3. Results and discussion

3.1. Statistical based modeling

3.1.1. Statistical evaluation

Table 2 shows the magnitude of main and interaction effects of operating variables on S_b, as a result of analysis of variance (*ANOVA*). It is worth mentioning that higher interactions between variables were negligible and therefore are not presented. Absolute estimates can be interpreted as deviations of the positive and negative settings from the mean of the respective factors. For instance, while the *superficial gas velocity* is switched from 1.1 (cm/s) up to 2 (cm/s), S_b will increase by an average of 28.9 s^{-1}. Fig. 1 presents 3D surface plots of the relationship between operating variables and S_b.

Table 2
Main and interaction effects of operating variables on S_b.

Variables	Absolute effects (s^{-1})	Relative effects (%)
J_g	28.9	54.41
ρ_{sl}	−10.8	20.33
C_f	6.9	12.99
T_f	4.3	8.10
Interaction ($J_g \times T_f$)	4.6	8.66
Interaction ($\rho_{sl} \times T_f$)	−3.7	6.97
Interaction ($J_g \times \rho_{sl}$)	2.8	5.27
Interaction ($J_g \times C_f$)	−1.5	2.82
Interaction ($C_f \times \rho_{sl}$)	0.2	0.38
Interaction ($C_f \times T_f$)	0.1	0.19

According to the results, the most important variable for S_b is *superficial gas velocity*, which produces a 28.9 s^{-1} increase in S_b. This is a result of much more bubble surface formed which offset the effect of bubble size increase. The next most significant variable is *slurry density (solids%)* which has a negative effect on S_b, with decrease of −10.8. This can be mostly attributed to bubble coalescence owing to elevated slurry viscosity and frother adsorbed on the surface of fine particles (due to significant amount of fine particles in feed, 55–60% less than 45 μm). Relatively high mixing conditions of industrial columns can increase the probability of bubble collision and coalescence (Massinaei et al., 2009). Frother *dosage* shows a positive effect on S_b but less than above variables. Due to minor effect of *frother type* on S_b, this parameter was not involved in modeling procedure as an input variable. Detailed discussions regarding the effect of operating variables on S_b can be found elsewhere (Massinaei et al., 2009).

3.1.2. Model development

General form of the empirical model to predict S_b as a function of *superficial gas velocity* (J_g), *slurry density (solids%)* (ρ_{sl}) and *frother dosage* (C_f) in industrial rougher columns can be expressed mathematically as

$$S_b = f(J_g, \rho_{sl}, C_f) \quad (2)$$

Various linear and non-linear models were fitted to data and for each model residual analysis was performed by plotting predicted vs. observed values by STATISTICA 6.0. In each statistical test, a new model expression was proposed and then the estimated and observed values of S_b were linearly correlated and statistical parameters were calculated. The performance of each model was evaluated by correlation coefficient (R) and root mean square error (*RMSE*) obtained from the following expressions:

$$R = \frac{n(\sum XY) - (\sum X)(\sum Y)}{\sqrt{\left[n\sum X^2 - (\sum X)^2\right]\left[n\sum Y^2 - (\sum Y)^2\right]}} \quad (3)$$

$$RMSE = \left[\frac{1}{N}\sum (X - Y)^2\right]^{\frac{1}{2}} \quad (4)$$

where X = predicted values, Y = experimental values and N = number of datasets.

The coefficients and exponents of the mathematical models were determined by a numerical method, called Quasi–Newton (Uribe–Salas et al., 1999; Cilek and Umucu, 2001; Cilek and Yilmazer, 2003). Finally, the model with the highest values of the correlation coefficient between observed and predicted S_b values was chosen as the target model as follows:

$$S_b = 102.44(J_g)^{0.56}(\rho_{sl})^{-0.38}(C_f)^{0.09} \quad (5)$$

Standard units of operating variables were used in the model, viz. cm/s for *superficial gas velocity* (J_g), % for *slurry density* (ρ_{sl}) and ppm for *frother dosage* (C_f). Statistical analysis of regression model comprising ANOVA table along with standard errors and confidence intervals of regression coefficients are listed in Tables 3 and 4. Relatively high standard deviations of some coefficient values are mainly due to experimental errors (random and systematic) in addition to natural fluctuations in feed composition and operating conditions in full-scale flotation circuits. In the authors opinions, gathering more experimental data, regular calibration of the used instrument (e.g. gas/slurry/reagent flow meters, pH meters, level controllers, etc.) by standard samples and colleting