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Towards a mechanistic model for slurry transport in tumbling mills

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ABSTRACT

A new modelling approach to slurry transport in dynamic beds based upon combining space and time-averaged Navier–Stokes equations with a new type of cell model is described. The resulting Ergun-like equation is used to correlate pressure drop with time-averaged distributions of the porosity, superficial fluid velocity and solids velocity for data derived from positron-emission-particle-tracking (PEPT) experiments in a scaled industrial tumbling mill fitted with lifter bars, pulp lifters and a discharge grate and run with particles and re-circulating slurry.

1. Introduction

Tumbling mills play a crucial role in the South African minerals industry. Given that SAG mills account for some 60% of operating costs, even incremental increases in efficiency hold the promise of enormous savings. To achieve improved efficiency it is natural to seek a better understanding of the processes taking place inside the mill. Considering the case of slurry transport which is of crucial importance both because it is the means of extracting the mill product and because slurry pooling can seriously degrade grinding efficiency. Unfortunately, the lack of understanding of this complex, inefficient transport mechanism is the main bottleneck in trying to improve grinding circuit efficiency (Songfack and Rajamani, 1999).

The development of positron-emission-particle-tracking (PEPT) has enabled the non-invasive observation of model mills under operational conditions (Parker et al., 1993). Using particles labelled with β^+ emitters (the tracer), the subsequent annihilation of the positron with an electron produces 511 keV back-to-back gamma pairs. Detection of a few such pairs provides the raw data for tracking the in situ flow field of the tracer through opaque environments. Large particle tracking gives information about the charge while small particle tracking yields data about the fluid flow. Through PEPT one can build up time-averaged velocity, porosity and shear rate distributions – the key drivers to flow in tumbling mills. The obvious difficulty associated with measuring, and hence modelling, these physical flow parameters have limited its value to the minerals industry in favour of empirical models. Consequently, many models are based on unrealistic assumptions. Hogg and Rogovin (1982) assumed that transport is limited to the slurry pool and that the interstices in the charge are always saturated. Morrell and Stephenson (1996) constructed a more usable transport model through both the slurry pool and charge using extensive measurements on a pilot scale ball mill fitted with discharge grates. Latchireddi (2002) extended the Morrell and Stephenson model to include a wider range of discharge grates and added the pulp lifter design. Both these models are built on extensive data bases and have thus gained wide usage in those operations falling within their window of design. However, both appear decoupled from the physical parameters known to drive flow through porous networks – their extrapolation potential is thus unclear. Moys (1986) employed a semi-mechanistic approach based on the Blake–Kozeny equation according to Bird et al. (1960). The use of empirical relations to obviate complicated integration coupled with the uniform porosity and static bed assumptions make the model less applicable in cataracting environments where charge dynamics and dilation dominate.

A wide body of literature is also devoted to the understanding of the porosity and viscosity – the key parameters in any fundamental fluid mechanics descriptions of transport through packed beds.

Mineral slurries exhibit non-Newtonian behaviour (see Shi and Napier-Munn, 1999; Klimpel, 1984). Beyond this conclusion, the literature becomes inconsistent as to the nature of the slurry (see Klimpel, 1984; Kawatra and Eisele, 1988). The combinatorial effect of solids concentration, temperature, chemical environment, particle size and distribution and their response to particle density, interactions and composition has sustained the ambiguity on rheological characterisation (Shi, 1994). The characterisation of viscosity is usually given by the slope of the experimentally constituted rheogram (shear stress versus shear rate curve), however, very little

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is known on the operating shear rate range of the tumbling mill system – a pertinent issue for the non-linear rheograms that typify mineral slurries. The experimental limitations associated with settling, especially for dense mineral slurries, have relegated most rheogram descriptions to high shear rate ranges. The authors have found no quantitative information on mill operating shear rate ranges in the literature and conclude that much of the published work is derived for ranges dissimilar to tumbling mills; see Section 4 for further details supporting this conclusion.

From the early work of Darcy (1856), on viscous flows to the standard model of flow through unconsolidated static beds (Ergun, 1952), there is no denying the overwhelming dependence of flow on porosity (Evans and Civan, 1994). In tumbling mills, the aggressive, opaque environment prevent apriori the use of direct measurement, favouring non-invasive imaging techniques like high speed video (Santomaso et al., 2003), bi-planar X-ray imaging (Govender et al., 2004) and PEPT (Parker et al., 1993). The spatial distribution of the porosity in tumbling mills has been investigated both experimentally by Parker et al. (1997) and numerically by Yang et al. (2003, 2008). While the calculation from the numerical modelling is based on explicit knowledge of each particles position and size, and therefore clear, the PEPT technique uses the residence time fractional distribution (Wildman et al., 2000; Wildman and Parker, 2002), as a proxy for the packing density with the assumption that the time-averaged calculations from a single particle are representative of the bed at steady state. Parker et al. (1997) found this distribution to be non-uniform for slowly rotating drums without lifter bars. The discrete element model used by Yang et al. (2003, 2008), in their numerical work also found a porosity distribution across the bed for slow moving mono-sized spheres. Multi-component mixtures and their influence on porosity distribution have been indirectly investigated up to binary mixtures by Yu and Standish (1987, 1991), Baumann et al. (1994), Dury and Ristow (1999). However, to the best of our knowledge, no work has been done on the porosity of multi-component mixtures of solid and liquid in tumbling mills.

On the theoretical side, a key issue for the construction of a mechanistic model is the correlation of observables. A conventional starting point for the discussion of slurry transport is the Ergun equation (Ergun, 1952). One must bear in mind, however, that the Ergun correlation between pressure drop Δp per unit length ΔL, porosity ϵ and superficial flow velocity U for a (Newtonian) fluid with viscosity μ and density ρ, and spherical particles of specific area a_v, was obtained by combining the Carmen–Kozeny and Burke–Plummer equations, and strictly applies to static packed beds; outside of that setting it comes with many caveats (Zoltani, 1992).

Indeed, the solids bed in slurry transport is dynamic rather than static. Yoon and Kunii (1970) in a simple experiment involving downward flow of solids at $\epsilon = 0.37$ against upwards gas flow, demonstrated that $\Delta p/\Delta L$ is a function of the so-called slip velocity, $U_{\text{slip}} = U - \epsilon V$, where V is the solids velocity. Yoon and Kunii suggested to merely replace U with U_{slip} in the Ergun equation, and in fact in the circumstance of fluidized beds the usual practice (see e.g. Mabrouk et al., 2007) is to follow this rule of thumb up to porosity $\epsilon = 0.8$.[1] Yet there is no real foundation for such a shortcut since the underlying capillary models are dubious at the higher porosities of eventual interest. Moreover, while an attempt can be made to extend the derivation of the Carmen–Kozeny equation to dynamic beds by averaging over moving capillary segments, such approaches fail in that they lead to the wrong dependence on superficial and solid velocities. One is thus compelled to a different approach.

This paper describes the first efforts to obtain a correlation for dynamical beds. In this the focus will be on viscous flow since there one has better theoretical control (the turbulence picture implicit in the Burke–Plummer term of Ergun's equation is subject to considerable doubt (Bear, 1972)). For the same reason a restriction to Newtonian fluids is made.

Section 2 describes a fresh attack on the dynamic bed problem: spatial averaging, which is a long standing approach to porous media (Whitaker, 1999), is combined with time averaging as familiar in the treatment of turbulence (albeit here the motivation is different). This leads to a formal relationship between pressure drop and drag on the (moving) solids in the flow.

To realise the drag relation in a practical way, we use a new cell model based upon cell averaging. This approach has the advantage of automatically giving the correct dependence on superficial fluid and solid velocities, as well as being free from arbitrary adjustable parameters and applicable at all porosities.

In Section 3, a description of the PEPT experimental program is given while the methodology for obtaining key inputs to the new cell averaging model is presented in Section 4. Section 4 also gives a quantitative comparison between the new model and the Ergun approach. Finally our conclusions and directions for future investigations are done in Section 5.

2. Pressure drop model

As a matter of principle all information regarding fluid flow may be had from the solution of the incompressible Navier–Stokes equations. In practice this is rather too much detail. Moreover the information concerning the influence of the solids on the flow is only implicit in the no-slip boundary conditions at the solid surfaces.

In turbulence theory one smears over detail via time averaging on a scale T:

$$\overline{\vec{u}(t,\vec{r})} = \frac{1}{T}\int_t^{t+T} dt'\vec{u}(t',\vec{r}). \tag{1}$$

On the other hand purely spatial averaging is a frequently used approach to stationary flow through static packed beds (see Whitaker, 1999, and references therein). Writing the fluid volume spatial average as

$$\langle\vec{u}\rangle = \frac{\epsilon}{V_f}\int_{V_f} d\vec{r}'\vec{u}, \tag{2}$$

and noting that in the case of the dynamic bed one is not concerned with short time-scale fluctuations so that it is useful to average over both space and time, we can define

$$\vec{U} \equiv \overline{\langle\vec{u}\rangle} \tag{3}$$

and similarly $P \equiv \overline{\langle p\rangle}$. We emphasize that such definitions are particularly appropriate to PEPT experiments, since in the interpretation of PEPT data one invokes the ergodic hypothesis for the trajectories of individual tracer particles.

In averaging to the continuity equation for an incompressible fluid, $\vec{\nabla}\cdot\vec{u} = 0$, one encounters a term $d\epsilon/dt$, where ϵ is the average porosity. This source term represents that in the dynamic bed a net flow of solids into or out of the averaging volume V during the time interval T is possible. On sufficiently long time scales one expects the system to become stationary. One can then omit the source term, yielding

$$\vec{\nabla}\cdot\vec{U} = 0 \tag{4}$$

[1] Fluidization commonly employs a different correlation at higher porosity (Wen and Yu, 1966); as $0.4 \leqslant \varepsilon \leqslant 0.7$ in the mill so there will be no occasion to use the Wen–Yu equation.