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Fractal evaluation of particle size distributions of chromites in different comminution environments

A. Taşdemir *

Eskişehir Osmangazi University, Department of Mining Engineering, Mineral Processing Division, 26480 Eskişehir, Turkey

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ABSTRACT

Particle size distributions (PSDs) are often rendered as cumulative functions, either as number of particles larger than a certain diameter, or as mass smaller than a certain diameter. The fractional exponent of the number/mass-size power law has been interpreted as the fractal dimension of the distribution. An application of PSD in comminuted chromites by means of the fractal mass distribution is presented. The five types of chromite samples were subjected to four comminution events; jaw, cone, hammer crushing and ball milling. The PSDs generated by different comminution devices has been evaluated by mass-based fractal fragmentation theory and the fractal dimensions of fragmentation (D_F), a value quantifying the intensity of fragmentation, have been obtained for each chromite ore. The results of the present study show that the particle size distributions of the comminuted chromites having different mineralogical characteristics are fractal in nature. Single and multifractal methods have been successfully applied to characterize particle size distributions (PSD) of chromite samples comminuted by different comminution devices. In general, depending on the energy events, the chromite ores having different mineralogical characteristics showed a general trend of PSDs, and hence, the ranges of D_F for a specific device. It can be concluded that breakage mechanisms have more effective on fractal dimensions of chromite samples although the mineralogical properties and size of the chromite ores broken are also a factor.

1. Introduction

Quantitative understanding of the evolution of particle size distribution is a necessary step for appropriate process control and optimization of comminution operation. Comminution can be defined as the process by which materials are reduced in size. Comminution processes such as crushing and grinding are essential stages in mining and mineral processing operations to reduce the size of ore and rock, and to liberate the valuable mineral for beneficiation (Sadrai et al., 2006). Many statistical methods have been proposed to describe particle size distribution (PSD) of comminuted materials and the most important PSD functions have been reviewed by Allen (1997). These include normal, log-normal, Gates–Gaudin–Schuhmann, Rosin–Rammler distribution functions, etc. Truncated size distributions are also used sometimes when a particle population would have every particle smaller than a definite top size (King, 2001). The most common truncated distribution is the logarithmic distribution and the other truncated versions of Rosin–Rammler, log-normal and logistic distributions can be generated. Fractal size distribution is another approach to make the size distribution curve a line (Ahmed and Rrzymala, 2005).

Mandelbrot (1982) noticed that several natural phenomena could be well described by a single power law. The exponent of this power law, called fractal dimension, is a non-negative real number that can assume fractional or non-fractional values. Fractal geometry was proposed with the objective being to better describe very irregular forms that are too complex to be described by Euclidean geometry. Fractals are self-similar objects (Mandelbrot, 1982). Such scale-invariant systems offer new opportunities for modeling the propagation of multiple fractures at different length scales. Because of their complexity at any given scale, they are particularly applicable to fragmentation and comminution of homogeneous and heterogeneous materials, and a fractal fragment size distribution is expected (Turcotte, 1992; Carpinteri and Pugno, 2003).

In order to understand fractal geometry, it is important to remember that in Euclidean geometry, a point has zero, a line one, a plane two and a volume three dimensions. The dimension (D) of a fractal is not necessarily an integer as in Euclidean geometry. In fact, it is a fraction with a range between the values of Euclidean geometry for a line and volume, i.e. the fractal dimension varies from 0 to 3 for true fractal sets. A value of fractal dimension higher than 3 cannot be explained physically (Bartoli et al., 1991). Consequently, a property described by a power law may not necessarily lead to a fractal behavior, since the exponent of this power law is not restricted to any particular value.

* Tel.: +90 222 239 37 50x3421; fax: +90 222 239 36 13.
E-mail address: atasdem@ogu.edu.tr

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The size-frequency distribution of particle size obeys a power law; hence, this scaling relationship can be characterized by its fractal dimension, a parameter simply drawn from the power law exponent (Lu et al., 2003). If the size of the fragments is a fractal, then the fractal dimension could be estimated from the size distribution of the fragments. Most applications of fractal concepts to PSDs are based on the fragmentation model of Turcotte (1986). PSDs have been analyzed with power-law functions relating cumulative number of particles to diameter and mass of particles to diameter, and the exponents interpreted as fragmentation fractal dimensions (D_F).

In this model, the fragmentation of an initially intact particle into smaller particles leads to a power-law relation between (i) number or (ii) mass of particles as a function of particle size. These two types of fragmentation relations are known as number-based and mass-based approaches (Turcotte, 1992). The fragmentation model does not lead to a geometrical fractal with the fractal dimensions confined between Euclidian dimensions. The sorting of particles by size in the fragmentation model results in fractal dimensions ranging theoretically between the limits of 0 and 3 (Bittelli et al., 1999).

D_F was used as a tool to understand the size reduction events in comminution in the past. The fractal dimension, characterizing the particle size distribution with a single parameter that retains most information, has become a useful tool in quantifying brittle material fragmentation (Thomas and Filippov, 1999). Based on the experimental ball mill studies, Zeng et al. (2002) comminuted a coal and the ground products on different grinding time were characterized by fractal particle size distribution. They concluded that the fractal dimension of PSD of the product at different grinding time was similar, which denoted that the fractal dimension of PSD can be used to characterize the self-similarity of coal comminution. It was shown by Piscitelle and Segars (1992) that PSD plays an important role in determining the fractal dimension of a material when using gas adsorption techniques. A nickel sulphide ore was subjected to two comminution events, impact shattering and ball milling and the applicability of fractal analysis in the resulting comminution products was carried out by Brown et al. (1993). It was shown that the fractal dimension of particle size distribution could be used to monitor the comminution capability and estimate the degree of particle comminution (Cui et al., 2006). Fractal dimension of PSD was used for describing the fineness of the comminution product by them.

The fragmentation mechanism induced by the comminution process used and the initial properties of the materials may be effective on D_F at a given size range. Several theoretical models have been proposed linking fractals to fracture and fragmentation. Recently, the fractal fragmentation theory to give a multi-scale interpretation of energy size effect have been developed and applied by Carpinteri and Pugno (2002) and Carpinteri and Pugno (2003). Assuming the fractal law for the size distribution of particles, they have unified the three comminution laws proposed by Rittinger, Kick and Bond for predicting the energy consumption in fragmentation. They also confirm the fractal nature of fragmentation and lead to the determination of the model parameters. The multifractality was explained by them as due to two different fracture mechanisms. The proposed theory emphases how the energy dissipation in the comminution process occurs in a fractal domain intermediate between a surface and a volume. For finer comminution D_F is close to 2 indicating the damage occurs in small concentrated zones, for larger particles it is close to 3 explaining the damage is more spatially distributed.

This work represents evaluation of fractal fragmentation dimensions obtained from particle size distributions (PSDs) of the chromite samples comminuted in different comminution devices (jaw, cone and hammer crushers, and ball mill). The aim of this study has been also focused whether or not the D_F is the same or different for comminuted products of chromite ore samples having different mineralogical properties when the ores are comminuted in the same and different devices. The mass-based fractal PSD model was fitted to 25 mass-size distributions obtained by comminuting and sieving of five types of chromite ores. Also, piecewise fractal model was applied to hammer crusher and ball mill products.

2. Materials and methods

2.1. Mineralogical properties of chromite ores used

The five types of chromite ore samples obtained from the run-of-mines, namely Bantlı which belongs to Karaburhan in Eskisehir, Turkey; Dereboyu, Kef, Lasir and Yunuskuyu which belong to Guleman in Elazig, Turkey are used in the studies. Chemical analysis of chromites by XRF is given in Table 1. A detailed mineralogical examination on the thin and polished sections of the lump samples of chromites for the texture of chromite and gang minerals was made and also the representative samples of chromite ores were examined by XRD. Measurements of Feret diameter which is the distance between two parallel planes measurements were made on the polished sections of unbroken ore samples by image analysis. Mean of mean Feret (μ) (*Mean Feret*, d_F: average of 8 Feret length measurements at 8 different angles,) and standard deviations (σ) of each samples were determined and the statistical parameters presented elsewhere (Taşdemir, 2008).

According to the results evaluated with petrographic/mineralogic examinations, XRD patterns and unbroken size distributions, the properties of the chromite ore types used are summarized as following:

Bantlı ore: The banded ore type consists of dominantly serpentinized olivines as the gang mineral. Chromite grains are mainly seen as cataclastic texture due to tectonic effects. Fractured chromite crystals are filled with serpentines. Serpentine is composed of sieve texture which contains olivine relicts. Based on the 141,404 measurements, the statistical parameters of grain sizes in the unbroken ore were found as μ = 87.54 μm and σ = 70.74 μm.

Dereboyu ore: The spotted ore type has mainly pyroxene mineral as the gang mineral. Pyroxene is serpentinized in the contact zone of chromites grains and fills the fractures. Chromite grains are fractured and show the cataclastic texture. Based on the 26,378 measurements, the statistical parameters of grain sizes in the unbroken were found as μ = 114.23 μm and σ = 129.89 μm.

Kef ore: The massive type of chromite has mainly unaltered olivine minerals. The alteration products are serpentine and chlorite minerals. The chromite grains within the unaltered olivine keep their original shapes and less fractured than the grains within

Table 1
XRF analysis of chromite ores used

Compound wt.%	Chromite ores				
	Bantlı	Dereboyu	Kef	Lasir	Yunuskuyu
MgO	30.0	18.5	20.0	20.0	28.0
Al_2O_3	3.1	9.9	17.0	9.7	3.3
SiO_2	23.2	9.3	7.4	14.1	24.3
CaO	0.3	0.5	0.3	1.1	0.3
Fe_2O_3	8.6	13.9	12.7	11.6	7.0
Cr_2O_3	25.0	45.2	39.8	41.3	30.3
LOI[a]	9.8	2.65	2.75	2.05	6.8
Total	100	100	100	100	100

All samples have Na_2O (%) = 0.1; P_2O_5 (%) = <0.1; K_2O (%) = 0.1; TiO_2 (%) = 0.1; MnO (%) = 0.1.
[a] Loss of ignition.