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Spreadsheet-based simulation of closed ball milling circuits

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Abstract

Grinding, particularly at finer sizes to achieve the required degree of liberation or specific surface, is a critical unit operation in terms of energy consumption and process optimization. Nowadays, considering the significant developments made in computer hardware and software, using simulation programs to optimize design and operation of various ore treatment plants could be very beneficial to the mineral process engineers. Currently, there are a number of steady-state simulators that can be run under DOS™ or WINDOWS™ environments such as BMCS, MODSIM™ and JKSimMet™. In this paper, COMSIM, a new simulator which runs under Excel spreadsheet will be introduced which uses a Population Balance Model (PBM) to simulate ball mills. Plitt and Nageswararao models have been used to describe the performance of hydrocyclones, which by linking to the ball mill model allow for closed-circuit simulations. To describe material flow through the ball mills, two tanks-in-series models called Weller's model and one plug flow unit plus *n* perfect mixers units (1PF + *n*PM) model have been used. Utilizing Microsoft™ Excel™ capabilities such as graphics and VBA programming to implement new macro-based user functions, the authors developed an easy-to-use comminution simulation environment. By comparing the outputs of COMSIM with measured grinding data and previously existing simulators, particularly BMCS, its performance was firmly tested both in terms of accuracy and precision of obtained results.

Keywords: Comminution; Grinding; Simulation; Spreadsheet; Modelling

1. Introduction

Grinding process mathematical modelling and computer simulation particularly in mineral industry had been a very active research field for several past decades. This is due to the high consumption of electrical energy in comminution circuits and low efficiency of operating mills especially in fine size range. The first attempts were made by Bond (1952) who introduced work index concept and developed an energy-based model of grinding mills as follow:

$$W = 10 \times W_i \left[\frac{1}{\sqrt{P_{80}}} - \frac{1}{\sqrt{F_{80}}} \right] \quad (1)$$

where W is the specific energy for grinding in kW h/ton, W_i is the work index in kW h/ton, P_{80} is the mill product d_{80} size in μm and F_{80} is the mill feed d_{80} size μm.

Given W, W_i and F_{80}, one can calculate the product size, namely, P_{80}. This is one of the simplest models to predict the ground size. Traditionally, Eq. (1) has been the main tool to design new mills, even though it can only predict one point of the product particle size distribution, P_{80}. Then, during 1960–1990, Population Balance Modelling (PBM) approach was extensively used to simulate grinding process, capable of predicting the full product particle size distribution. Recently, high fidelity simulation (HFS) approach which collectively refers to a number of modelling methods such as discrete element modelling (DEM), computational fluid dynamics (CFD), discrete grain breakage (DGB) and finite element methods (FEM) has been the subject of many new researches in mineral processing (Herbst et al., 2002). In this paper, the authors explain

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the empirical models and population balance model which used in building the COMSIM (comminution simulator) Software. Nowadays, most steady-state grinding simulators are based on phenomenological models of milling devices, as a type of population balance models which their parameters must be determined from plant samplings. Excel™ as the spreadsheet of choice by most engineers because of its popularity, computational and graphical capabilities. For this reason, the authors decided to implement mathematical model of tumbling mills and hydrocyclones in this environment. Creating new functions using Visual Basic for Application (VBA) is an important feature of Excel which is not normally utilized by users. In COMSIM, various macros were defined by VBA to provide simulator functionalities which are not supported by Excel.

2. Ball mill modelling

Population balance modelling tumbling mills includes developing rate–mass balance relationships which are inherently phenomenological and their parameters must be determined by running experimental tests. Hence, simulation of industrial tumbling ball mills requires a lot of effort for calibration of general mathematical model of ball mills to a specific plant. Briefly, these efforts consists of plant sampling campaigns, laboratory tests to determine breakage function parameter and mill feed and discharge particle size analysis, selection function estimation, model validation and finally performing simulations.

2.1. Grinding process model

The continuous grinding process model is based on the system of batch linear equations of grinding as follow (Luckie and Austin, 1972; Prasher, 1987):

$$\frac{\mathrm{d}m_i}{\mathrm{d}t} = -S_i m_i + \sum_{j=1}^{i-1} b_{ij} S_j m_j \quad \text{for } i = 1, 2, \ldots, n \tag{2}$$

where m_i, S_i and b_{ij} are mass on ith screen, selection function of ith size class and non-cumulative breakage function of ith size class when particles are broken from jth parent size class, respectively. The matrix form of solution to this system of equations is:

$$\underline{m}(t) = T \exp[-St] T^{-1} \underline{m}(0) \tag{3}$$

where $\underline{m}(t)$ is particle size distribution vector at time t. T is a square transformation matrix used to solve the system of Eq. (2) and can be calculated employing the following recursive algorithm:

$$T = \begin{cases} 0, & i < j \\ S_j, & i = j \\ \frac{1}{S_i - S_j} \sum\limits_{k=1}^{i-1} b_{i,k} S_k t_{k,j}, & i > j \end{cases} \tag{4}$$

$b_{i,k}$ in Eq. (4) describes the progeny size distribution after primary breakage of a single particle which has been explained in detail in next section. Inverse matrix of T, i.e. T^{-1}, can be computed by numerical algorithms when matrix T is found or can be computed directly based on a recursive algorithm similar to Eq. (4). By integrating system Eq. (3) over residence time distribution (RTD) of particles passing through the mill, the continuous form of the grinding model can be obtained. In COMSIM, the following matrix forms of the mathematical model of continuous tumbling ball mills (using Weller's and one plug flow plus n perfect mixers residence time distributions) has been implemented. Both models are available to the user:

(a) Using Weller's model as residence time distribution function:

$$\underline{m}_\mathrm{d} = T[I + S\tau_\mathrm{s}]^{-2}[I + S\tau_\mathrm{l}]^{-1} \exp[-S\tau_\mathrm{pf}] T^{-1} \underline{m}_\mathrm{f} \tag{5}$$

where $\underline{m}_\mathrm{d}$ is the mill discharge particle size distribution vector, T is the square transformation matrix, S is the selection function diagonal square matrix, τ_s, τ_l and τ_pf are the Weller's model parameters and $\underline{m}_\mathrm{f}$ is the mill feed particle size distribution vector.

(b) Using one plug flow plus n perfect mixers model as residence time distribution function:

$$\underline{m}_\mathrm{d} = T[I + S\tau]^{-n} \exp[-S\tau_\mathrm{pf}] T^{-1} \underline{m}_\mathrm{f} \tag{6}$$

where τ and n are the mean residence time in each perfect mixer and the number of perfect mixers in model, respectively. Determination of RTD model parameters is a prerequisite to simulate a grinding circuit which will be explained in more details in Section 2.4.

2.2. Breakage distribution function

Breakage distribution function (also known as breakage function and appearance function) is a fundamental parameter of tumbling ball mill model. It must be determined experimentally by performing breakage tests on representative samples of ore. There are several methods to obtain the breakage function of ore; such as direct methods including drop weight and twin pendulum tests and indirect methods by grinding a powder sample of ore in laboratory mills. The authors have developed a windows program called BFDS (Breakage Function Determination Software) which computes the breakage function according to laboratory grinding data (Yousefi et al., 2005).

2.3. Selection function

This is another parameter of tumbling ball mill model and in fact is a measure of the grinding kinetics taking place inside a mill. Selection function or specific rate of breakage depends on a number of factors such as particle