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Impact model for micrometer-sized sand particles☆

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ABSTRACT

Predicting sand erosion and deposition in aero-engines requires accurate prediction of the coefficient of restitution of micron-sized sand particles impacting surfaces in the gas path. The paper presents a framework for sand collision modeling including the effects of elastic, elastic–plastic and plastic deformations, surface adhesion, and size dependent property variations of sand. Based on the Stronge model, a modified recovery stage model is proposed and validated with experiments. The proposed recovery model is shown to be more accurate in predicting COR compared to the Stronge and Jackson–Green models. The proposed model also considers the large sensitivity of the mechanical properties of sand to the grain size. By using available experimental data in the literature, it is shown that the Young's modulus increases significantly when the particle diameter decreases from 1 mm to 0.1 mm, and increases gradually with a further decrease in size. It is also shown that the effective yield stress increases dramatically for particle sizes under 100 μm. The proposed model is compared to experimental measurements for 150 μm and 20–40 μm sand particles impinging on steel and aluminum surfaces at velocities up to 90 m/s and at different incidence angles. Overall the predictions are in good agreement with the experiments and fall within the experimental data spread of $\pm 1\sigma$.

1. Introduction

Particle-surface interactions occur widely in many natural and man-made systems in the chemical, pharmaceutical, power, and aerospace industries. These interactions result in wear and erosion, deposition, and in some cases corrosion of the surface. The mechanics of impact varies with the particle and surface material properties, impact speed, size and shape of particle among other factors. For example, ingested sand or volcanic ash in aero-engines can lead to severe erosion of the compressor blades changing their aerodynamic characteristics and increasing the propensity for dynamic stall. These particles can also be drawn into the hot flow path, melt or soften and deposit on the nozzle vanes and blades blocking cooling circuits, eroding thermal barrier coatings, and in many instances leading to corrosion and pitting. The mechanics of how the particles erode, deposit, clog cooling circuits and damage the thermal barrier coating are not well understood because of the micron sized particles, high speed, and high temperatures. Therefore, an understanding of particle impact behavior is a significant step towards elucidating on erosion and deposition in many industrial applications. This paper focuses on the interaction of a micrometer-sized particle with a surface.

Many analytic models have been investigated to describe and understand the impact and contact process of particles, and can be divided into two kinds of models, discrete models and continuous model [1]. The former assumes that the collision occurs in a short time and the configuration of impacting bodies does not change significantly, and is confined primarily to rigid bodies. Continuous models describe the contact forces and deformation, and will give a better description of the real behavior of the collision, such as spring-dashpot model [2], Jackson–Green (J–G) model [3], and Stronge model [4]. Typically, collision is characterized by the coefficient of restitution (COR), which is a measure of the energy lost during impact. Energy losses during collisions are described by losses attributed to plasticity, viscoelasticity, adhesion, friction, and other dissipative mechanisms, such as elastic waves and breakage [5,6]. Nominally, the COR is dependent on the properties of the impacting materials, on the velocity and angle of impact, and on material surface characteristics such as relative roughness and friction. Electrostatic, thermophoretic, and capillary forces could also play an important role in many applications. Adhesion due to van der Waal's force plays a dominant role in energy loss for micron-sized particles at low impact velocity [5,7], whereas plastic deformation losses dominate at high impact velocities [8].

Most impact models have been based on principles of energetics and mechanics guided by experiments. Greenwood and Williamson (GW) [9] used Hertz theory to model elastic impact between two contacting spheres. Tabor [10] divided the process of impact into three stages; elastic deformation, plastic deformation, and rebound, considering energy losses during each stage to calculate the coefficient of restitution, and showed that the recovery or rebound stage of the impact is reversible and essentially elastic. While Tabor's approach

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Nomenclature

a	contact radius (mm)
b_1 b_2	constant coefficient of yield stress
C_R	surface roughness factor
C_H	Hertz damping coefficient
E	Young's modulus (GPa)
E_*	Effective Young's modulus of particle and target (GPa)
e	coefficient of restitution
e_n	normal COR of impact
e_t	tangential COR
F_n	normal contact force (N)
f_0	circumferential tension of the adhesion force per unit length (N)
l	characteristic length of sand particle (μm)
m_*	effective mass of particle and target (kg)
P	impulse (N·s)
p	contact pressure (MPa)
R	radius of particle (μm)
R_*	effective radius of particle and target (μm)
R_*^r	effective radius during recovery stage (μm)
V	Velocity (m/s)
W_A	work of adhesion forces (J)
W_{diss}	work of dissipative forces (J)

Subscripts

c	subscript of critical value between elastic and elastic–plastic stages
cp	subscript of critical value between elastic–plastic and full plastic stages
t	tangential
n	normal

Greek alphabet

α	impact angle (degree)
γ	surface free energy (J/m^2)
Δ	average prediction error of COR
δ	deformation displacement (μm)
δ_{max}	maximum displacement (μm)
δ_r	recovery displacement (μm)
μ	friction coefficient
ν	Poisson's ratio
σ_Y	yield stress (MPa)
$\tilde{\sigma}_Y$	predicted effective yield stress (MPa)
σ_{Y0}	macroscopic yield strength (MPa)

was that of macroscopic surface physics, Johnson [7] considered elastostatics, elastic impact of spheres, oblique impact of spheres, wave propagation during an impact, and plastic impact at moderate speeds, among other factors. Chang, Etison and Bogy (CEB) [11,12], improved the GW model by including plastic deformation beyond the elastic limit. This model showed better coefficient of restitution predictions when compared to results from Tabor's and Johnson's models. Thornton [13] investigated the collision by dividing the impact into a perfectly elastic and perfectly plastic phase, providing an analytical relation for coefficient of restitution as a function of normal impact velocity. Li et al. [14] modified the Johnson's model [7] to include more detailed load variation and presented a theoretical model for coefficient of restitution for the normal impact of a rigid sphere. The proposed contact force-displacement relations and restitution coefficient predictions showed good agreement with finite element analysis. Wu et al. [15] investigated the impact of an elastic sphere with elastic and elastic–plastic surface for only finite plastic deformation using finite element method (FEM). The study concluded that dissipation due to stress wave propagation was negligible compared to dissipation due to plastic deformation. Weir and Tallon [16] also proposed an equation to predict the coefficient of restitution for normal particle impacts at lower velocity. This study predicted that the coefficient of restitution for equally sized sphere–sphere impact to be 19% smaller than for sphere–plate impacts. Vu-Quoc and Zhang [17] presented an elasto-plastic normal force displacement model for spheres in collision, but the model requires input from FEM simulations.

While the above studies focused on the mechanistic aspects of impact of relatively large particles, adhesive forces become dominant at microscales and low impact velocities. Bowling [18], discusses the variety of forces such as the van der Waals force that contribute to adhesion. The adhesion force often is quantified through the use of an adhesion energy which is distributed over the contact surface of the two bodies in contact. It is assumed that all the adhesion energy required to separate the particle from a surface is lost in the separation process. The JKR and DMT theories [19,20] have been extensively applied in adhesive contact problems [21–24]. For micrometer sized particles, Singh and Tafti [25,26] combined the Jackson–Green (J–G) model [3] for elastic, elastic–plastic collision with the adhesion model proposed by Brach and Dunn [27] based on JKR theory. The adhesion model was invoked during the recovery stage of the collision to obtain a combined COR. They validated their model with experiments of Tabakoff [28] for particle sizes of 150 μm and those of Reagle et al. [29] for particle sizes between 20 and 40 μm. They combined this model with a thermal model based on critical viscosity [30] to calculate the probability of deposition for sand particles in a turbine blade cooling duct with rib turbulators [31].

In spite of developments in modeling collision dynamics of micron sized sand particles, there are still disagreements with experiments and room for improvement. While some of the disagreements can be contributed to the challenge of measuring the COR for micro-sized particles, the impact modeling has many underlying assumptions. While the elastic part of the collision is well characterized by Hertz theory, the plastic stage of the collision is semi-empirical in nature, and so is the recovery stage. The other contributing factors are uncertainties in the material property of sand (dependent on composition and size of grains) and the characterization of surface energy for the adhesion losses. Unfortunately, there are few studies on micrometer-sized sand particle collision dynamics and property characterization. Oliver [32,33] introduced a method for measuring hardness and elastic modulus for small scale materials by indentation techniques. Using this technique, Dutta and Penumadu [34,35] measured the Young's modulus and hardness of Ottawa sand grains. Similar work was also done by Daphalapurkar et al. [36] to measure the Young's modulus of sand using nano indentation method. McDowell [37] tested single grains and aggregates of silica sand, and verified that the yield stress could be approximated to be one-fourth of the average strength. Brzesowsky [38] studied the failure stress of single sand grains ranging from 115 μm to 378 μm, by translating the critical force at failure into the failure stress using Hertz theory. These studies have established the dependence of Young's modulus and yield strength of sand on size and composition.

In his paper we develop a model for COR calculation by proposing a new collision recovery model and by including the effects of sand grain size. The model is validated with experiments. The paper is organized as follows: First, the contact models for elastic, elastic–plastic and plastic compression stages are given, followed by a recovery model based on Stronge [4]. A new elastic recovery model is then proposed with molecular adhesive forces acting on the contact area. The combined modified model is then compared to experimental results for particles of size 0.1 to 1.6 mm. Finally, the model is extended to micron-sized sand particles by considering the effect of scale on the material properties of sand, and validating against available experiments.