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Axial dispersion of granular material in inclined rotating cylinders with bulk flow: Geometric model for 50% fill

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ABSTRACT

Monte Carlo particle tracking methods used to model particle motion in inclined rotating cylinders with bulk flow have been reported in the literature. This paper reports some analytic results for the mean squared deviation of axial position in the following special case: the particles are in the rolling mode; the bed surface is flat, with no axial variation of bed depth; the avalanche is instantaneous and of negligible thickness; collisional axial dispersion is ignored; fill = 0.5. Fill = 0.5 means that in every half-rotation of the cylinder every particle will have participated in one and only one avalanche; this simplification allows analytic results to be obtained. The calculations are carried out by relating the number densities of tracer particles in successive cycles of avalanching. In the case of well-mixed avalanches, the model is also developed using a random walk approach. The results confirm predictions obtained from discrete element modelling and Monte Carlo particle tracking simulations, which indicate that the mean squared deviation of axial position can oscillate with time for short enough times.

1. Introduction

Rotary kilns are widely used in industry; see, for example Boeteng [1]. An understanding of axial dispersion in rotating cylinders is of practical relevance since it controls the residence time distribution of the particles. Monte Carlo particle tracking methods have been used to model particle motion in inclined rotating cylinders with bulk flow [2–7]. The general picture of particle motion is that a particle rotates with the cylinder, essentially as part of a rigid body rotation, until it reaches the free surface of the bed of particles. Then it falls down the free surface in an avalanche, and is reabsorbed into the rigid body rotation. During the rigid body rotation a particle experiences little change in axial position, but during an avalanche there is axial movement driven by the inclination of the cylinder. The avalanching leads to two contributions to axial dispersion: collisional, caused by collisions between the particles, and geometrical, caused by there being a distribution of axial distances travelled in an avalanche. In horizontal cylinders with no bulk flow only collisional axial dispersion occurs.

In situations where there is bulk flow, Campbell [8] has shown that it is important to use a measure of mean squared displacement which incorporates the effects of the mean flow. Here, the measure of axial dispersion which will be used is the mean squared deviation relative to the mean axial displacement of the bed, *RMSD* [9],

$$RMSD = \left\langle (z(t) - z(0))^2 \right\rangle - \langle (z(t) - z(0)) \rangle^2 \tag{1}$$

where z is axial position. This will be used here, though in particle tracking models all particles can start at $z = 0$, so the terms with $z(0)$ in Eq. (1) are unnecessary.

Third et al. [10] have modelled axial dispersion of granular material in inclined rotating cylinders with the granular material in the rolling mode; see e.g. Mellmann [11] for a review of the forms of motion. In the rolling mode there are continuous avalanches down the free surface of the bed, and in a cross section the surface of the bed is approximately flat. The motion of a particle is as described earlier, with a fall down the free surface in an avalanche after which the particle is reabsorbed into the rigid body rotation, which rotation transports the particle back to the avalanching region. Third et al. [10] modelled axial dispersion of approximately monosized particles in an inclined rotating cylinder with bulk flow in the special case of the bed surface being flat with no axial variation of bed depth, by two methods: (i) the discrete element modelling (DEM) and (ii) Monte Carlo particle tracking calculations. In the Monte Carlo particle tracking model, the following simplifying assumptions were made: the avalanches were of negligible duration and thickness. It was found that for short enough times, *RMSD* oscillated with

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time. Third et al. [10] argued that the oscillations were a consequence of geometric axial dispersion.

This paper shows that in the case of fill $f = 0.5$, analytical solutions can be found for *RMSD*. The calculations are carried out by relating the number densities of tracer particles in successive cycles of avalanching. In the case of well-mixed avalanches, the model is also developed using a random walk approach. Fill $f = 0.5$ means that in every half-rotation of the cylinder every particle will have participated in one and only one avalanche, which simplification allows analytic results to be obtained. It has not proved possible to obtain analytic results for fills $f \neq 0.5$.

The results from this very simple model are not expected to be realistic. In particular, collisional dispersion, the thickness of avalanches and the duration of avalanches are important. However, the approximations were made to demonstrate the effect of the geometry on mixing, and it is hoped that the results are indicative of real behaviour. The DEM calculations of Third et al. [10] show that the geometric axial dispersion considered here can be significant at short enough times. The results here help to clarify the dynamics of axial particle displacements, and may be useful in checking Monte Carlo particle tracking methods.

2. Particle tracking model

A cylinder of radius R, with axis inclined at angle β to the horizontal, rotates about its axis at constant rotational speed, and a bed of particles flows through the cylinder. The angle of repose of the particles is γ. The size of the particles is assumed to be negligible compared to the size of the cylinder. The surface of the bed is flat, the bed is of uniform depth and the cylinder is half-full.

The surface of the bed is shown in Fig. 1, where bulk motion is from left to right. A particle leaves the region of rigid body rotation and appears on the surface at A. The particle travels in an avalanche down the free surface along the line of steepest descent, and is reabsorbed into the rigid body rotation at point B. For simplicity, the avalanche is taken to be instantaneous and of negligible thickness, and collisional axial dispersion is ignored. The line AB makes an angle α with the cross section of the cylinder, where α is given by [12–15]

$$\tan\alpha = \frac{\sin\beta}{\sqrt{\sin^2\gamma - \sin^2\beta}}. \tag{2}$$

The axial distance Δz that the particle travels in the avalanche is given by

$$\Delta z = \tan\alpha(r_A + r_B). \tag{3}$$

The initial condition has N tracer particles distributed uniformly over a narrow axial slice of the bed at $z = 0$, so the initial concentration, that is number density, of tracer particles is

$$c_0(r,z) = \frac{2N}{\pi R^2}\delta(z) \tag{4}$$

where $\delta(z)$ is the Dirac delta-function and r is the radial coordinate, so that

$$\int_{r=0}^{R}\int_{z=-\infty}^{\infty} c_0(r,z)\pi r dr dz = N. \tag{5}$$

Within the particle bed, c_0 is independent of azimuthal coordinate. After half a rotation of the cylinder, all the particles will have rotated to the free surface, travelled down the free surface in an avalanche, and been absorbed into the rigid body rotation. No particle will have undergone a second avalanche. Because (i) the time of travel down the free surface has been neglected, and (ii) the transit time of a particle rotating through the bed from the lower free surface to the upper free surface is independent of r, the concentration of tracer particles after half a rotation of the cylinder, c_1, will be independent of azimuthal coordinate, but will be a function of r and z.

The reason for this azimuthal symmetry is as follows. The surface of the bed is shown in Fig. 2. Half a rotation of the cylinder will be called a cycle, with period T_c. The material entering element E comes from the avalanche formed from the shaded part of the surface of the bed. If at the start of a cycle the bed is azimuthally symmetric, the composition of material reaching the shaded part of the surface of the bed is the same throughout the cycle. Thus the material entering element E is the same throughout the cycle, and thus bed is azimuthally symmetric at the end of the cycle. The same symmetry will exist after each subsequent cycle.

This symmetry allows straightforward analytical calculations to be performed. The symmetry does not exist at fills other than 0.5, because in that case the duration of a period of rigid body rotation depends on radial position.

The concentration of tracer particles at the end of the nth cycle, where n is an integer, will be denoted by c_n; it is independent of azimuthal coordinate.

Two cases will be considered: well-mixed avalanches, and avalanches with no mixing.

2.1. Well-mixed avalanches

An equation enabling c_{n+1} to be found from c_n will now be developed. Consider the tracer particles which avalanche from the shaded

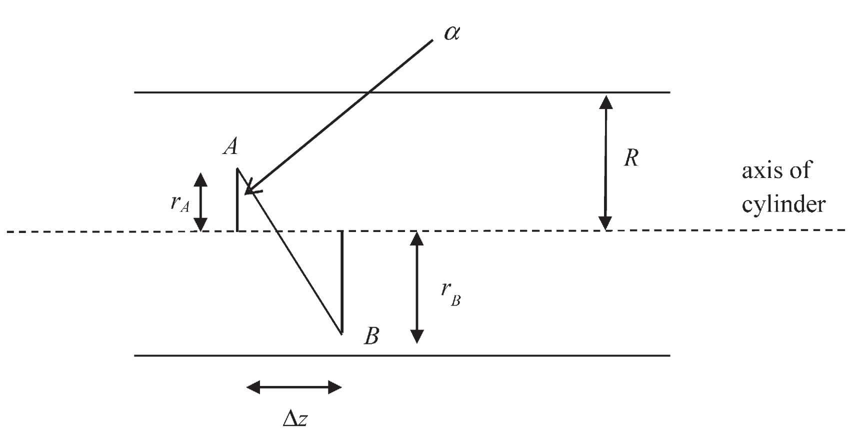

Fig. 1. View of the surface of the bed of particles. Bulk flow is from left to right. The rotation brings particles to the surface in the upper part of the figure and removes particles from the surface in the lower part of the figure. The line *AB* shows the trajectory of a particle during an avalanche.