Powder Technology 287 (2016) 121–130

Contents lists available at ScienceDirect

Powder Technology

journal homepage: www.elsevier.com/locate/powtec

An application of breakage mechanics for predicting energy–size reduction relationships in comminution

Chunshun Zhang [a,*], Giang D. Nguyen [b], Jayantha Kodikara [a]

[a] Department of Civil Engineering, Monash University, Clayton, VIC 3800, Australia
[b] School of Civil, Environmental and Mining Engineering, The University of Adelaide, Adelaide, SA 5005 Australia

ARTICLE INFO

Article history:
Received 29 July 2015
Received in revised form 18 September 2015
Accepted 26 September 2015
Available online 30 September 2015

Keywords:
Grain size distribution
Energy–size reduction relationship
Breakage mechanics
Grain crushing

ABSTRACT

We propose a new continuum approach to predicting the energy–size reduction relationship in comminution. The approach is based on a breakage mechanics model that accounts for the evolution of the grain size distribution (GSD) due to comminution with only five parameters, all of which are physically meaningful. The model is implemented and used in finite element analysis to study the energy–size reduction relationship in a roller mill. Predictions of product GSDs and energy consumption are validated against experimental counterparts. On this basis, a series of parametric studies in terms of various roller gaps and grinding speeds are carried out in order to explore: 1) the optimum roller gap, in which the maximum new surface can be created while the least input work is required; and 2) the influence of increasing grinding speeds on new surface creation.

1. Introduction

Although a great deal of effort has been focused on studying the relationship between energy consumption and grain size reduction, the application of these studies for improving the efficiency of industrial comminution processes is still limited. As a consequence, in practice the energy required to grind minerals still accounts for a very small amount of the total energy consumed in a typical grinding operation (e.g., approximately 4% [35]). Conventional formulas such as the three grinding laws [7,25,29] were initially proposed to describe the energy–size reduction relationship of a single particle. Those methods were later considered to account for the grinding process of granular assemblies of various sizes through the integration of the sizes of particles. However, none of those classic approaches actually works as expected [18]. A typical problem is the selection of an adequate (reference) grain size. Within a gradation of mineral particles, there is no one exclusive size. For different grain sizes taken as the criteria, the energy–size relationship would be very different (i.e., different for sizes D_{10}, D_{50} or D_{80}, being the sieve sizes on which 10%, 50% and 80% of the overall sieved mass retained, respectively), that results in each of these models working in a different and limited energy–size range.

Rigorous models derived from population balance principles (PBM) [28] provide another approach to deal with particles of various sizes, which are underpinned by mass balance. Through the introduction of new concepts such as selection functions and breakage functions, prediction of the energy–size reduction relationship was improved (e.g., [17,22–24]). However, the parameters required in those functions are generally empirically fitted to product grain size distributions (GSDs) and do not conform to the laws of thermodynamics (energy balance and positive entropy production). Besides, Liu and Schönert [19] proposed another interesting pragmatic model taking into account the interaction of particles of different sizes via an energy split function to distribute energy to different size fractions contained in a particle bed. This improves the application of PBM by providing another balance equation for energy, in addition to the equation for mass in PBM. Nevertheless, the approach used strong assumptions on the independence of breakage function from the feed particle size distribution and the negligible energy loss due to friction.

It is understood that comminution is quite complex, with the fracture and fragmentation of grains, followed by the flow of grains and their fragments. The input energy is therefore not only consumed by the grain fracturing process, but also dissipated through friction, acoustic emission, heat transfer and redistribution of strain energy [27]. As such, neither the conventional approaches nor PBMs are well capable of describing the energetics of the comminution process.

An interesting way to deal with granular comminution is to use Discrete Element Method (DEM). Whereas DEM allows a detailed study of the micro-dynamics of particles, they are generally computationally intensive, which results in a limit either in the duration of a simulation or in the number of particles [2,8,9]. The results are highly sensitive to minor perturbations in the initial conditions and easily affected by numerical noises. Moreover, it is difficult to determine and calibrate the constitutive force–displacement relationship describing the contacts between grains. In overcoming the above-mentioned DEM issues, Finite Element Method (e.g. [20,37]) can be used to study

* Corresponding author.
E-mail address: ivan.zhang@monash.edu (C. Zhang).

http://dx.doi.org/10.1016/j.powtec.2015.09.040
0032-5910/

the micromechanical behaviour of a single or a few particles. However, its application to modelling crushing of assemblies of particles in roller crushers requires a capable constitutive model that can capture essential features of material crushing processes.

Our present work aims to offer a more mechanistic way of gaining insight into breakage distributions and energy consumption in comminution of assemblies of particles, which acknowledges internal classification in terms of grain size and location (the GSD varies spatially within the crusher). The product GSD will be predicted via the mechanical simulation in terms of the feed particle size distribution, the local mechanical properties of the particle assemblies, and the geometrical properties of the crusher relative to local grain sizes and position. For that purpose, comminution will be simulated using the newly developed theory of continuum breakage mechanics [13,14] implemented in the Finite Element Method (FEM) package ABAQUS. The unique feature of models based on breakage mechanics theory is that they all possess an internal variable characterising the crushing process of an assembly of particles, besides their capability to capture other associated mechanisms of dissipation during comminution (such as rearrangement of particle fragments, and inter-particle friction). Models based on breakage theory therefore possess intrinsic links between the evolving GSD and stress levels that will be useful for the simulation of comminution in a roller mill and the prediction of the evolving GSD during the passage of grains through the rollers. This theory along with a simple breakage model will be briefly described in Section 2. In Section 3, the breakage model is integrated in the ABAQUS FEM package and then used to predict product GSDs and consumed moment work of quartz sand passing through the roller mill. Finally, the energy–size reduction relationships are primarily explored in Section 4 under various rollers' gaps and grinding (rotation) speeds.

2. Brief reviews of breakage mechanics and a simple breakage model

2.1. Breakage mechanics theory

Recently, Einav [13,14] developed a novel continuum breakage mechanics theory under a thermodynamic framework to tackle various problems involving grain crushing. The theory incorporates the GSD and its evolution (through breakage, B) to capture the macroscopic behaviour of crushable granular materials. As a summary, the rigour and the novelty of the breakage mechanics theory can be attributed to five key points relevant to the current paper, as highlighted below.

(A) The breakage variable B is an internal variable representing the crushing degree of an assembly of particles (or more precisely, a representative volume element), via an evolving cumulative GSD ($F(D,B)$), which is measured from the initial ($F_0(D)$) and ultimate cumulative GSDs ($F_u(D)$):

$$F(D,B) = (1-B)F_0(D) + BF_u(D). \tag{1}$$

(B) The ultimate GSD, $F_u(D)$, determines the critical state, at which particles could not be further crushed. In the absence of near walls and boundaries, it is generally practical to employ a fractal power law $F_u(D)$ [26,31,36] to describe the GSD by mass. Fractal GSDs are understood to develop in the absence of nearby walls. Readers are referred to the above-mentioned literatures and many others (e.g., [4,10,26,31,34]) for further details and demonstration of the applicability of fractal GSDs. Therefore, *only in the absence of nearby walls* we chose to employ a fractal power law $F_u(D)$ in the following form:

$$F_u(D) = \frac{D^{3-\alpha} - {D_m}^{3-\alpha}}{{D_M}^{3-\alpha} - {D_m}^{3-\alpha}} \tag{2}$$

where D_m and D_M are the minimum and maximum sizes of the grains, and the fractal dimension α usually ranges from 2.5 to 3.0. In constrained large shear–strain problems like in a roller mill, the collision probability of a particle with others is independent of its size, and α appears to approach 3.0 [32]. In such cases Eq. (2) has a logarithmic limit:

$$F_u(D) = \log(D/D_m)/\log(D_M/D_m).$$

(C) The specific elastic strain energy stored in larger grains is greater than that in smaller grains. To a first order this energy scales with the grains' surface area. This assumption is based on the fact that bigger grains carry more contacts in proportion to their larger surface. It has been verified by Einav [13] and Ben-Nun and Einav [2] using DEM under various loading conditions. The success of breakage mechanics models in various applications [6,11,27,39] proved the validity and practical usefulness of this assumption.

(D) Through statistical homogenization, the macroscopic specific elastic strain energy is the average of such energies in the various size fractions, and can be determined from the current GSD, $F(D, B)$. Further details can be found in Einav [13,14].

(E) Breakage dissipation during comminution is equal to the loss in the residual breakage energy (the residual breakage energy reflects the energy left in the system to crush particles towards the ultimate GSD, $F_u(D)$).

Various breakage models have been developed, considering the characteristics of linear and nonlinear elasticity, hydro-mechanical effects of partial saturation, chemo-mechanical effects on degradation, finite deformation, porous compaction and dilation. Relevant applications of those various models have been reported in Nguyen and Einav [27] and Das et al. [11] on geophysics, Zhang et al. [39] on piled foundation, Buscarnera and Einav [6] and Zhang and Buscarnera [40] on unsaturated soils, Rubin and Einav [30] on porous granular material, Buscarnera [5] on chemo-mechanical degradation; and Einav and Valdes [16] on mechanics of granular mixtures. In this paper, we employed the simplest breakage model based on linear elasticity. For brevity, only constitutive equations of the model are summarised below. Readers can refer to Einav [13,14] and Nguyen and Einav [27] for further details.

2.2. A simple breakage model

In this study a simple breakage model based on breakage mechanics theory [13,14] will be presented and used for the analysis. This is to provide sufficient background and details for the descriptions of the numerical modelling and research results. The model describes the material behaviour as a continuum, while possessing variables that characterise the evolution of internal particle-to-particle processes such as particle crushing and rearrangement of fragments, friction between fragments. This is an advantage over traditional continuum models based on plasticity theory, while having the computational efficiency of a continuum model compared to discrete approach like DEM. For isothermal process, the stored macroscopic specific elastic strain energy of a RVE (representative volume element) consisting of several particles is described by:

$$\Psi = (1-\vartheta B)\left(\frac{1}{2}K\varepsilon_v^{e^2} + \frac{3}{2}G\varepsilon_s^{e^2}\right) \tag{3}$$

where K and G are the bulk and shear moduli; ε_v^e and ε_s^e are the elastic volumetric and shear strains in triaxial conditions. The grading index ϑ measures how far the initial cumulative GSD is away from the ultimate one.