Powder Technology 284 (2015) 326–335

Contents lists available at ScienceDirect

Powder Technology

journal homepage: www.elsevier.com/locate/powtec

Dependence of wall stress ratio on wall friction coefficient during the discharging of a 3D rectangular hopper

Pengkai Xu, Xuezhi Duan, Gang Qian, Xing-Gui Zhou *

State Key Laboratory of Chemical Engineering, East China University of Science and Technology, Shanghai 200237, PR China

ARTICLE INFO

Article history:
Received 11 October 2014
Received in revised form 16 May 2015
Accepted 28 June 2015
Available online 3 July 2015

Keywords:
3D rectangular hopper
DEM
Wall friction coefficient
Wall stress ratio
Modified effective internal friction angle
Force network

ABSTRACT

Discharging of a 3D rectangular hopper is simulated by Discrete Element Method (DEM) to understand the influence of wall friction on wall stress ratio. The role of the side and face walls in supporting the particle load by friction is interpreted by force network analysis. With the increase of wall friction coefficient, the relative density of the force chains attaching the walls increases, which enhances the contribution of the walls to supporting the particle load. The enhanced supporting of one wall due to wall friction will weaken the supporting of its neighboring wall. However, the formed force chains attaching one wall will exert normal stress on its neighboring wall. When the wall friction coefficient is large enough, the strength of the relative strong force chains attaching the walls eventually relies on the internal friction coefficient and the length of the hopper along the normal direction of the walls, making the wall stress ratio less sensitive to wall friction coefficient. The wall stress ratio is predicted with Walker's model, in which the effective internal friction angle is replaced by a modified effective internal friction angle to relate to the wall and internal friction coefficients. The prediction is more accurate than with the assumption of critical equilibrium state.

1. Introduction

Hoppers are widely used in industry for protection, storage and delivery of solids, and frequently have cylindrical [1,2] or rectangular shapes [3–5]. Compared to cylindrical hoppers, rectangular hoppers are more attractive in view of efficient space utilization and low fabrication cost [4]. However, the flow behavior of the solids in rectangular hopper is more complicated because of the geometric asymmetry [4]. When designing hoppers, safe operation is the most important issue to be considered in charging and discharging. Improperly designed hoppers may collapse while discharging, leading to disastrous consequences.

The wall normal stress is a critical variable for safe hopper design. It is related to particle and hopper properties, which is described in the Janssen equation [6,7]:

$$P_w = \frac{\rho_b g D_h}{4\mu_w}\left[1-\exp\left(\frac{-4\mu_w kz}{D_h}\right)\right] \tag{1}$$

where P_w is the wall normal stress, k is the wall stress ratio and D_h is the hydraulic diameter of rectangular hopper. As demonstrated by the Janssen equation, wall normal stress is scale dependent. Therefore, the wall normal stress obtained in small hoppers cannot be used for the design of large scale hoppers [3,8,9]. Compared to wall normal stress, wall stress ratio, a dimensionless parameter calculated by dividing the wall normal stress by the vertical normal stress, is less scale dependent, especially for hoppers with sufficiently large sizes. From the wall stress ratio, the wall normal stress can be calculated by Janssen's equation. Hence the wall stress ratio is an important parameter for hopper design.

For the prediction of wall stress ratio, Balevičius et al. have proposed a model with assumption of negligible effect of wall friction [7]. However, because the wall friction changes the stress distribution in the granular material near the wall, it changes the wall stress ratio [10]. Walker's model [7], which relates the wall stress ratio to the macroscopic wall friction angle and effective internal friction angle, has theoretically integrated the effect of wall friction, indicating that the wall stress ratio changes monotonically with wall friction coefficient. While the macroscopic wall friction angle is related to the microscopic wall friction coefficients, the effective internal friction angle, which is based on the assumption of critical equilibrium state that bulk solid tends to slip, is independent of the microscopic wall friction coefficients [7,11]. This makes the prediction inaccurate because the solids are seldom in critical equilibrium state before entering into the converging flow zone of hoppers [12]. Moreover, the stress ratio of one wall may also be influenced by the friction coefficient of its adjacent walls, because the load transmitted to one wall could be influenced by its adjacent walls [13]. However, this problem has not yet been addressed.

Recently, Discrete Element Method (DEM) is frequently employed for an in-depth understanding of solid flow behaviors because the simulation based on DEM is reliable and includes dynamic behaviors. A good fit between simulation with DEM and experiments was observed for the wall normal stress during charging and discharging of a rectangular

---

* Corresponding author. Tel.: +86 21 64253509; fax: +86 21 64253528.
E-mail address: xgzhou@ecust.edu.cn (X.-G. Zhou).

http://dx.doi.org/10.1016/j.powtec.2015.06.054
0032-5910/

hopper [7]. The arch structure in the force network shown by DEM simulation [14] was reproduced by experiments [15,16], and the exponential decay of the normal force distribution indicted by DEM simulation was consistent with the experiments [17]. On the basis of the transient behaviors of the force network, Masson and Martinez related the wall normal stress with the particle properties [18], and Zhu and Yu provided an outline of the flow regions in hopper discharging [1].

The multiscale information from DEM also makes it possible to understand how wall fiction coefficient affects wall stress ratio. Masson and Martinez determined the wall stress ratios in a 2D hopper when the internal friction coefficient was 1 and the wall friction coefficients were 0.5 and 1 [18]. Landry et al. determined the wall stress ratios in a 3D cylinder hopper when the internal friction coefficient was 0.5 and the wall friction coefficients were 0.5 and 2 [19]. Both of these studies show significant changes of the wall stress ratio with the wall friction coefficient. Therefore, for a reliable prediction of wall stress ratio, wall friction coefficient has to be considered. However, the relation between the wall friction coefficient and the wall stress ratio has not been established, and the mechanism how the wall friction coefficient affects the wall stress ratio has not been revealed.

In this work, we focus on the effect of the wall friction coefficient (i.e. side wall friction coefficient and face wall friction coefficient) on the wall stress ratio in discharging a 3D rectangular hopper. A possible mechanism is proposed by analyzing the force networks in the hoppers with different wall friction coefficients. Finally, the effective internal friction angle for the calculation of wall stress ratio is modified to relate to the wall and internal friction coefficients. This avoids the use of the critical equilibrium state assumption and makes the prediction more reliable.

2. Numerical method

2.1. DEM simulation

The rectangular hopper and particle system under investigation are shown in Fig. 1 with the hopper geometries and particle properties [20] listed in Table 1. The hopper heights are 0.54 m and 0.62 m for the hoppers with widths of 0.10 m and 0.15 m, respectively, and the particle number changes between 12,800 and 22,100 accordingly. To understand how the stress ratio of one wall depends on the friction coefficients of its adjacent walls, we have considered the conditions that the friction coefficients of side and face walls are different in this work.

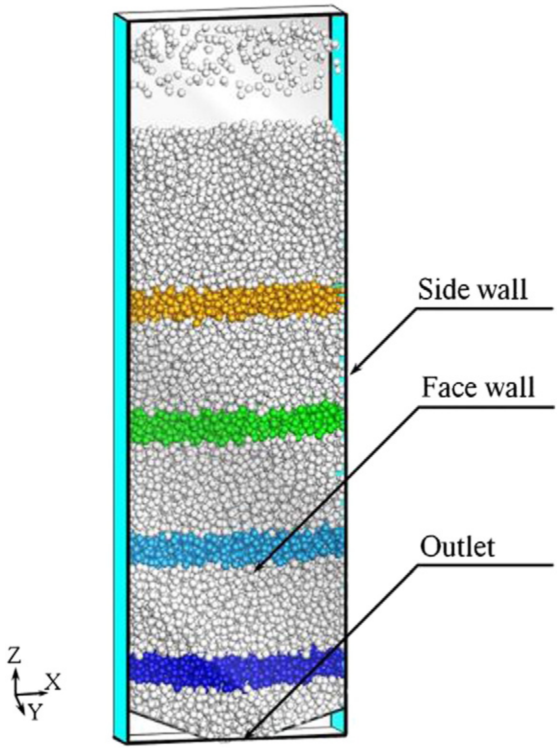

Fig. 1. Geometry of the model hopper.

Table 1
The properties of particles and geometrical parameters of the hopper [20].

Quantity	Value
Particle diameter (mm)	6
Particle density (kg/m^3)	964
Particle number	12,800–22,100
Time step (s)	1×10^{-6}
Normal damping coefficient (N/(m/s))	
Particle/particle	1.39
Particle/wall	2.78
Tangential damping Coefficient (N/(m/s))	
Particle/particle	0
Particle/wall	0
Normal spring coefficient (N/m)	
Particle/particle	345,000
Particle/wall	690,000
Tangential spring coefficient (N/m)	
Particle/particle	345,000
Particle/wall	690,000
Friction coefficient particle/particle	0.52
Hopper width (m)	0.10, 0.15
Hopper height (m)	0.45, 0.54
Hopper thickness (m)	0.054, 0.072, 0.09, 0.10
Hopper half angle (deg)	60°
Hopper outlet width (m)	0.03
Side wall friction coefficient	0.2–0.5
Face wall friction coefficient	0.1–0.5

Simulation of the hopper discharging by 3D DEM involves the following steps: the generation of particle system, the detection of particle/particle and particle/wall contacts, the calculation of force acting on each particle, and the update of translational and rotational motions with time [21].

Monodisperse particles are generated to fill the 3-D hopper via the *En masse* method [22]. The particles we used are spherical and non-cohesive, which are frequently encountered in industries [23]. The governing equations of motion in the model are two momentum conservation equations:

$$m\frac{\partial \vec{v}_i}{\partial t} = \vec{F}_{gi} + \Sigma \vec{F}_{cij} \tag{2}$$

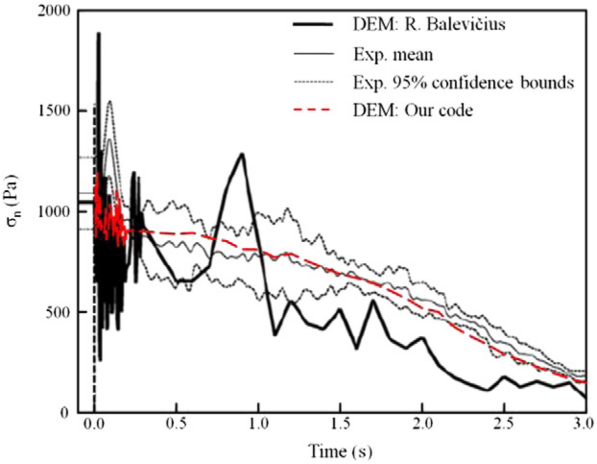

Fig. 2. Numerical and experimental results of wall normal stress vs. time in batch discharge of the flat hopper.