Powder Technology 283 (2015) 210–226

Contents lists available at ScienceDirect

Powder Technology

journal homepage: www.elsevier.com/locate/powtec

The extrapolation of the Drucker–Prager/Cap material parameters to low and high relative densities

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ARTICLE INFO

Article history:
Received 5 February 2015
Received in revised form 12 May 2015
Accepted 16 May 2015
Available online 23 May 2015

Keywords:
Powder compaction
Roller compaction
Modeling
Drucker Prager Cap
Extrapolation
Calibration

ABSTRACT

The present study explores new approaches to extract Drucker–Prager/Cap (DPC) constitutive model parameters at low and high densities of compacted powders for which it is not possible to get solid, undamaged samples for model calibration purposes. Extrapolations were carried out by invoking a number of physically plausible assumptions for high density conditions and the addition of the experimental shear cell testing procedure for low density extrapolations. The effects of these extrapolations on finite element model (FEM) results of both die compaction and roller compaction were examined. The sensitivity of the extrapolated DPC parameters on compaction model results was explored by performing parametric studies for both low and high density extrapolations. Examination of die compaction model results for low density showed little sensitivity to extrapolations; however, we are able to show that extrapolations of DPC parameters to low density may have a significant effect on roller compaction modeling results. High density die compaction FEM simulations reveal a significant effect on the way in which the DPC model parameters are extrapolated to high density. A method of extrapolating the DPC model parameters to high density is presented in this work. The work presented here demonstrates the significance of properly calibrating the DPC model at low and high densities and provides the necessary guidance for this purpose.

1. Introduction

Finite element based continuum mechanics modeling is a common tool used for predicting the behavior of powder material during compaction processes [1–5]. The most accepted phenomenological model for modeling the compaction of metal, ceramic, and more recently pharmaceutical powders, is the Drucker–Prager/Cap (DPC) plasticity model [6]. Owing to its popularity is the ability to calibrate the model from a small number of experiments. To obtain these experimental inputs, generally the calibration of the DPC model requires cylindrical flat-faced compacts over as large of a relative density range as possible. When implementing the DPC model into the finite element method, however, it is necessary to provide the material parameters for the full range of relative densities from the initial relative density, RD_0, up to the fully dense material, $RD = 1$. Experimental data can only be obtained for the range of relative densities within which intact specimens can be obtained, thus making it necessary to extrapolate the material parameters outside this range. These extrapolations are subject to the risk of producing results that may be inaccurate.

The objective of this paper is to (1) propose a methodology of extrapolating the DPC parameters to regimes that are not accessible experimentally so that the whole range of densities from RD_0 to $RD = 1$ is covered and to (2) evaluate the sensitivity of FEM compaction model results on the extrapolated parameters.

The proposed methodology is based on in-die results and the incorporation of shear cell experiments for the extrapolation of the failure surface DPC parameters to low density and the implementation of a porous plasticity model for the extrapolation of the cap surface DPC parameters to high density. In addition, we demonstrate for the first time, that extrapolation of DPC parameters at low density is extremely important for simulations of rolling compaction.

2. The DPC constitutive model

The DPC model provides an inelastic hardening mechanism that accounts for plastic deformation during compaction and volume dilatancy when the material yields in shear. Central to this model is the yield surface shown in Fig. 1, which is divided into two principal segments: a shear failure surface F_s that describes the behavior of the powder under low hydrostatic pressure, and a cap surface F_c that describes hardening behavior and densification of the powder. In the p–q plane, the shear failure surface is represented simply as a straight line and is defined by

$$F_s = q - d - p\tan(\beta) = 0 \quad (1)$$

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http://dx.doi.org/10.1016/j.powtec.2015.05.027
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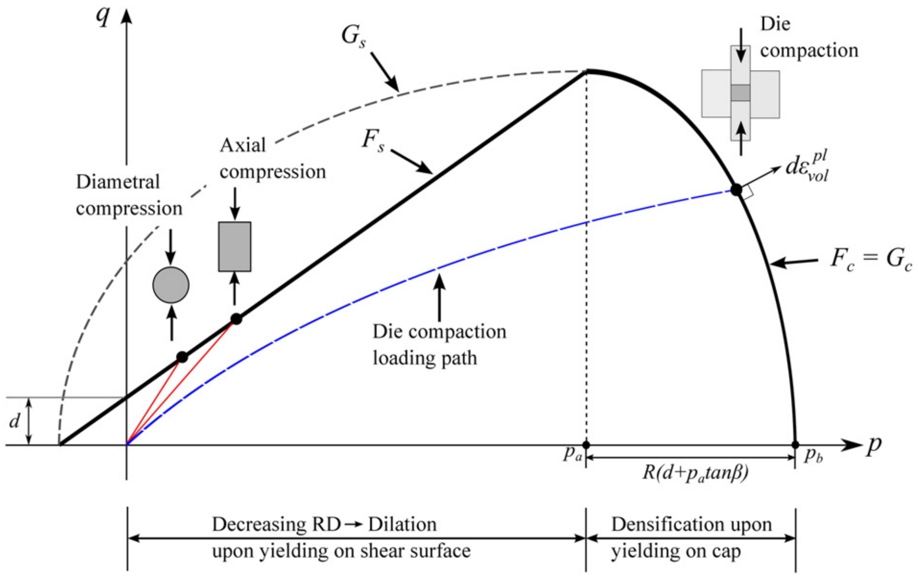

Fig. 1. Modified Drucker–Prager/Cap model: yield surface in p–q plane with experimental procedures for determining the shear failure surface F_s and the cap surface F_c.

where p is the hydrostatic stress, q is the von-Mises effective stress, β is the failure line angle, and d is the cohesion. The cap yield surface describing the densification of the powder is an ellipse given by

$$F_c = \sqrt{(p-p_a)^2 + (Rq)^2} - R(d + p_a \tan\beta) = 0 \tag{2}$$

where R is a measure of the eccentricity or shape of the ellipse, and p_a is the point along the p-axis that represents the intersection of the shear and cap surfaces and is termed the evolution parameter. As the material densifies the yield surface shown in Fig. 1 expands and the evolution of this expanding yield surface is described by the hardening law p_b as a function of the volumetric plastic strain $\varepsilon_{vol}^{pl} = \ln(\mathrm{RD} / \mathrm{RD}_0)$, where p_b is the hydrostatic yield stress. The four material parameters d, β, R and p_b are considered to be functions of the out-of-die relative density. A potential surface that dictates the plastic flow of the material under stress is also uniquely defined by these parameters and is associated on the cap and non-associated with the failure yield surface [6]. The overall flow potential is formed by two elliptical portions. The elliptical flow potential surface in the cap region is identical to the cap yield surface and is given by

$$G_c = \sqrt{(p-p_a)^2 + (Rq)^2} - R(d + p_a \tan\beta) \tag{3}$$

and the other non-associated elliptical portion in the failure surface region is given by

$$G_s = \sqrt{[(p_a - p)\tan\beta]^2 + q^2}. \tag{4}$$

The elliptical flow potentials G_c and G_s given by Eqs. (3) and (4) predict densification (increase in RD) and volume dilatancy (decrease in RD) respectively in the corresponding regions of the hydrostatic pressure above and below p_a.

2.1. DPC model calibration

A practical calibration of the constitutive model is achieved by the series of experiments shown in Fig. 1, where diametral compression, axial compression, and die compaction experiments are employed. For completeness we describe briefly the parameter identification of the DPC constitutive model. For more details see e.g., [1,7,8]. The calibration of the DPC model is achieved by the determination of the four independent parameters d, β, R, and p_b (p_a can be determined from these parameters), which can be obtained from a number of possible experimental procedures. The die compaction experiment requires a fully instrumented die so that both axial and radial stresses are measured, and allows for the determination of R, and p_b provided that d and β are known.

Cohesion d and internal friction angle β for a particular level of relative density are obtained via diametral and axial compression strength tests of cylindrical compacts. The diametral compression strength test is used to probe the tensile strength of compacts [9–12] and is often referred to as the "Brazilian disk test". The mathematical expression developed by Hertz [13] for thin elastic disks under point loading is commonly used for this measure and is given by

$$\sigma_T = \frac{2P_T}{\pi D t} \tag{5}$$

where P_T is the load at failure, D is the diameter of the cylindrical sample, and t is the thickness. The stress state in p–q space is given by $p = 2\sigma_T/3$ and $q = \sqrt{13}\sigma_T$ for diametral compression. Similarly, the axial compression strength test is carried out by axially pressing a cylindrical compact that provides the strength of the material in simple compression and is given by

$$\sigma_C = \frac{P_C}{A} \tag{6}$$

where P_C is the load at failure and A is the cross sectional area of the cylindrical compact. The stress state in p–q space for axial compression is given by $p = \sigma_C/3$ and $q = \sigma_C$. The cohesion and internal friction are defined in terms of diametral and axial compressive strengths by

$$d = \frac{\sigma_C \sigma_T \left(\sqrt{13} - 2\right)}{\sigma_C - 2\sigma_T} \tag{7}$$

$$\beta = \tan^{-1}\left(\frac{3(\sigma_C - d)}{\sigma_C}\right). \tag{8}$$

Diametral and axial compression strength tests require intact compacts that can be handled with little or no damage. This requirement