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A micromechanical study on the stress rotation in granular materials due to fabric evolution

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ABSTRACT

In the present study, the non-coaxiality between the axes of principal stress and strain rate tensors is investigated from micromechanical point of view. Based on the so called stress–force–fabric (SFF) relationship, which describes the macro–micro relation, an expression is derived for the principal direction of stress tensor in terms of micromechanical parameters of the fabric. In general, the rotation of principal stress axis and accordingly, the non-coaxiality angle, are influenced by both the anisotropy coefficients and directions of anisotropy of the fabric characteristics. The derived macro–micro relationship was evaluated by performing DEM simulations of 2D specimens of aggregates. It was shown that the principal directions of anisotropy parameters are almost coincident for the assemblies containing circular particles or elongated angular particles with random distribution. In such case, the principal direction of stress tensor can be regarded as the average principal direction of anisotropy. However, when the aggregate with elongated particles has inherently-anisotropic fabric, a correct estimation of the stress angle rotation requires considering all the anisotropy parameters including both anisotropy coefficients and directions.

1. Introduction

In the practical and conventional soil elastic–plastic constitutive models, it is generally assumed that the principal axis of stress coincides with that of the strain rate, i.e., the principle of coaxiality [1]. However, there are experimental and micromechanics-based observations to indicate that these principal axes do not coincide. In the soil mechanics literature, it is appeared that Roscoe et al. [2] was the first who reported the results from simple shear tests on sands concerning non-coaxiality of stress and strain rate tensors. Drescher & de Josselin de Jong [3] reported non-coaxiality in the deformation of an assembly of photoelastic disks in the simulation of two-dimensional granular media. High deviations of the axes were also observed in directional shear cell [e.g., 4,5], hollow cylindrical apparatus [e.g., 6–13] and plain strain (Schneebeli cylinder) tests [14]. In all the experiments, it was found that the deviation is significant at small shear strain, but gradually reduces with the increase in the shear strain and they coincide at large deformations. In addition, a change in loading direction may lead to an abrupt change in the non-coaxiality angle [e.g., 5,9,11,15]. Many attempts have been made to consider non-coaxiality effect in domain of constitutive soil modeling too [e.g., 16–22]. The conceptual reason of non-coaxiality can be explained by studying the micromechanical evolutions in the fabric.

In granular materials, it is obvious that the macroscopically observed behavior is in general a consequence of microstructural response at particle scale. In fact, the mechanical behavior can be well interpreted as a consequence of fabric evolution in the granular medium. The technical term 'fabric' describes spatial arrangement of particles, voids, and associated contacts. Based on experimental [e.g., 23–25] as well as numerical studies [e.g., 26–28], micromechanical investigations reveal that reciprocal mechanisms of generation and collapse of column-like microstructures among particles can explain the shear strength mobilization and deformational behavior of the aggregate media during the loading process. Hence, the deviation in the axes of stress and strain rate tensors can be described by the fabric evolution.

Microstructural evolution in a granular assembly depends on fabric anisotropy, which is distinguished by 'inherent' and 'induced' types. Induced anisotropy occurs during the loading process and shear deformation. However, inherent anisotropy is generally initiated during the deposition of soil particles under gravity so that the long axis of particles tends to align in a specific direction, which is termed as bedding plane. Using the so-called directional shear cell apparatus, Wong & Arthur [5] examined the effect of inherent anisotropy on the coaxiality behavior. They observed coaxiality along the plane of isotropy (the plane normal to the bedding angle), while non-coaxiality was clearly observed along other directions.

Many attempts have been made in order to quantitatively describe the fabric in a granular material. For instance, different forms of the so-called 'fabric tensor', which describes either the distribution of contacts among particles or the orientation of particles, were introduced

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[23,29–32]. Regarding the estimation of stress state in a granular assembly, Hill [33] defined the average stress tensor in terms of applied forces over a homogeneous granular system [see also 3,32,34,35]. Weber [36] introduced a macroscopic stress tensor, which can be calculated from assembly contact forces and the geometrical arrangement of contacting particles. Based on the Weber's equation, Rothenburg [35] showed that the average stress tensor for an assembly comprising circular particles or spheres has the properties of the stress tensor as used in the continuum mechanics, but is derived from consideration of discrete contact forces, contact geometry and principles of static equilibrium. He developed useful relationships for the assemblies with planar particles (circular disks), which equate the micromechanical parameters to the macroscale stress tensor of the system. By assuming that the distributions of average contact force components and contact normals have the same directions of anisotropy, the so-called stress–force–fabric relationship (SFF) was introduced [37] and its applicability was examined for the assemblies with circular [38], elliptical [39] as well as rigid and breakable polygonal particles [40,41], which were randomly distributed. Note that for the inherently-anisotropic assemblies containing elongated particles, however, this relationship is not applicable since the principal directions of contact force and contact normals among particles are not coincident anymore [28]. Li and Yu [42] explored the mechanism of non-coaxiality from the particle scale. They used directional statistical theory to study the anisotropy in the fabrics and characterized stress direction in terms of direction tensors. More recently, Seyedi Hosseininia [43] has introduced a general form of stress–force–fabric relationship for planar particles with arbitrary angular shape and fabric anisotropy. He generalized the Rothenburg's relationship by consideration of the normal and tangential components of the contact vector lengths with respect to the contact plane of two adjacent contacting particles. The proposed relationships were evaluated by performing numerical simulations of inherently anisotropic assemblies with polygonal elongated particle using Discrete Element Method (DEM).

Since DEM captures more detailed data about the inter-particle features, it has been adopted as a complementary tool to the experimental apparatus by which, the macro- and the micro-mechanical behavior of granular assemblies can be studied. Regarding the examination of coaxiality by DEM, Alonso-Marroquin et al. [44] observed non-coaxiality in a two-dimensional (2D) assembly of randomly distributed convex polygons. Another series of 2D DEM simulations were carried out by Thornton & Zhang [45] to study the shear banding and simple shear non-coaxial flow rules. They have reported a non-coaxial behavior similar to the experimental results of Roscoe et al. [2] and Roscoe [46]. Real tests on sands using a hollow cylinder apparatus [e.g., 9–13,15, 47] also showed that non-coaxiality is dependent on the anisotropy as well as the loading history. By using DEM and considering two-clumped circular disks as one rigid particle, Li and Yu [48] showed that the coaxiality assumption between the internal structure and the contact forces is not valid in the case of non-proportional loading on granular assemblies. They also showed that the simple form of Rothenburg's SSF does not work in such loading condition.

Apart from the particle scale viewpoint, non-coaxiality has been a main issue in constitutive modeling of granular soils from continuum viewpoint. The notion of non-coaxiality is the non-coincidence between principle stress direction and principle plastic strain increment direction. The physical origin of non-coaxial behavior in anisotropic granular media has been clearly identified to be the fabric anisotropy [49,50] and attempts have been made to provide rigorous formulations in the yield surface and flow rules in order to account for fabric effect [e.g., 49, 51–53]. When the formulated model is supplemented by an appropriate micromechanically calibrated fabric evolution law, the non-coaxial behavior in granular media can be convincingly explained and the non-coaxial material response can be predicted.

All the DEM works mentioned above have attempted to relate the existence of non-coaxiality to the anisotropic condition of the fabric in which, the relationship was described qualitatively. The objective of the present study is to investigate more accurately the effect of micromechanical parameters on the deviation of the directions of stress and strain rate axes within a granular material. Based on the generalized form of the micromechanics-based stress–force–fabric relationship [43], a general mathematical expression defining the direction of principal stress axis is derived. Hence, the relationship between fabric parameters and non-coaxiality can be described and discussed quantitatively rather than qualitatively. By using DEM simulations of a granular assembly, the applicability of the expression is examined by fixing the direction of strain rate axis and instead, the deviation of the principal stress axis direction from that of strain rate axis is investigated.

2. Stress–force–fabric relationship

In a granular assembly, the general expression of the Cauchy stress tensor related to microscopic average parameters can be written as follows [43]:

$$\sigma_{ij} = m_v \int_0^{2\pi} \left\{ \overline{f}_n(\theta)\overline{l}_n(\theta)n_i n_j + \overline{f}_n(\theta)\overline{l}_t(\theta)n_i t_j + \overline{f}_t(\theta)\overline{l}_n(\theta)t_i n_j + \overline{f}_t(\theta)\overline{l}_t(\theta)t_i t_j \right\} E(\theta)d\theta. \tag{1}$$

The term m_v is the density of contacts (the number of contacts per unit area). $\overrightarrow{n} = (\cos\theta, \sin\theta)$ and $\overrightarrow{t} = (-\sin\theta, \cos\theta)$ are the vectors representing the normal and tangential directions with respect to the contact plane between a pair of particles.

In the equation above, $E(\theta)$ indicates the portion of the total number of contacts in the medium, which is oriented at angle θ. The orientation of a contact is defined as the angle between the normal direction to the contact plane and the horizontal direction (see Fig. 1). According to Rothenburg [35], the distribution of contact normal orientation can be approximated by a second-order Fourier series expression:

$$E(\theta) = \frac{1}{2\pi}\left[1 + a_c \cos 2(\theta - \theta_c)\right] \tag{2}$$

where a_c describes the anisotropy in contact orientations and θ_c is the major principal direction of anisotropy. The parameter a_c represents the proportional difference in the number of contacts oriented along the major direction of anisotropy, i.e., $\theta = \theta_c$ and that in the perpendicular direction ($\theta = \theta_c \pm 90\,°$). In other words, if the distribution of

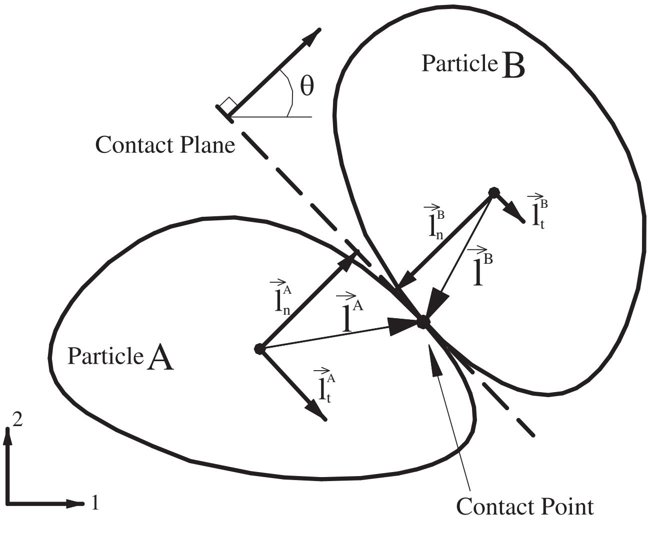

Fig. 1. Schematics of contact vectors and their decomposed components with respect to contact plane for two contacting particles A and B.