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A revisit of pressure drop-flow rate correlations for packed beds of spheres

Esra Erdim [a,1], Ömer Akgiray [b,*], İbrahim Demir [a]

[a] Istanbul Technical University, Faculty of Civil Engineering, Department of Environmental Engineering, Istanbul, Turkey
[b] Marmara University, Faculty of Engineering, Department of Environmental Engineering, Istanbul, Turkey

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ABSTRACT

A large number of correlations can be found in the literature for the calculation of pressure drop caused by fluid flow through packed beds. New correlations continue to be proposed and there appears to be no general agreement regarding which correlation is the most accurate. In this work, experiments have been carried out with water using glass spheres of nine different sizes, varying from 1.18 mm to 9.99 mm. For each size, experiments were repeated with at least two different porosities. A total of 38 correlations from the literature have been evaluated and a uniform notation was established to facilitate the comparison of the correlations. While the Ergun equation remains the most widely-used correlation, the data collected in this work shows that it should not be used above $Re_m \approx 500$. A simple new equation ($f_V = 160 + 2.81Re_m^{0.904}$) is proposed to represent the data collected in this work. The new equation yields the smallest mean error among all the correlations considered here. While a substantial amount of the data collected in this work involved D/d_P ratios less than 10, the correlations that fit the current data best do not have any wall effect correction terms.

1. Introduction

Flow of fluids through packed beds of solid particles occurs in a variety of important applications in several engineering fields. A quantity of primary interest is the pressure drop (or the head loss) generated as a result of fluid flow through the porous medium. The equation published by Ergun [1] about sixty years ago remains the most popular and the most widely-quoted pressure drop–flow rate relation for fluid flow through packed beds. For incompressible flow through a bed of spherical particles of identical size, the Ergun equation can be written as follows:

$$\frac{-\Delta P}{L} = 150\mu\frac{(1-\varepsilon)^2}{\varepsilon^3}\frac{V}{d_p^2} + 1.75\rho\frac{(1-\varepsilon)}{\varepsilon^3}\frac{V^2}{d_p} \quad (1)$$

where $-\Delta P$ = piezometric pressure drop in the bed, L = depth of the bed, V = velocity based on the empty cross-section of the bed, ε = porosity of the bed, d_P = particle diameter, and ρ = fluid density, μ = fluid viscosity. It may be noted that $-\Delta P = \rho gh$, where h = head loss in the bed.

In addition to a number of historically significant equations that predate it, many additional equations have been proposed since the publication of the Ergun equation and new correlations continue to be developed and published. Each new correlation is claimed to be more accurate than, or in some other way superior to the previously proposed correlations. However, there aren't many independent comparisons of these correlations. The few studies evaluating and comparing various correlations ignore most of the correlations in the literature and focus only on a few correlations. The purposes of this work can be outlined as follows: (1) Review all the widely-used and/or well-known correlations that exist in the relevant literature. (2) Present all the mentioned correlations using a uniform notation so that their application and comparison with each other are facilitated. (3) Test and compare the accuracy and the applicability of the correlations. Some recently proposed correlations have also been included in this evaluation.

In what follows, the definitions of various quantities of interest and the notation used in this work are first described. Correlations considered in this work are then rewritten using the described notation instead of the notation used by their original authors. Expressing correlations from different sources using a uniform notation will facilitate the comparison of the mentioned correlations and the observation of differences as well as similarities between the different equations.

Abbreviations: $-\Delta P$, piezometric pressure drop $-(P_2 - P_1)$; A, a characteristic area for the bed; A_x, the cross-sectional area of the empty bed; D, column diameter; d_P, particle diameter; ε, porosity of the bed; ε_b, bulk zone porosity; f, friction factor; f_k, friction factor ($f_k = 3f$); f_V, friction factor ($f_v = f_k\frac{Re}{1-\varepsilon}$); f_P, modified particle friction factor $\left(f_p = f_v\frac{(1-\varepsilon)^2}{\varepsilon^3 Re}\right)$; F_k, kinetic force exerted by the fluid on the solids; h, head loss in the bed; K, a characteristic kinetic energy per unit volume; L, depth of the bed; V, velocity based on the empty cross-section of the bed; V_b, bulk zone velocity; ρ, fluid density; μ, fluid viscosity; Re, Reynolds number ($\frac{\rho d_p V}{\mu}$); Re_m, modified Re number ($\frac{Re}{1-\varepsilon}$); Re_1, modified Re number ($\frac{Re}{6(1-\varepsilon)}$); T, tortuosity.

* Corresponding author. Tel.: +90 216 348 02 92/261; fax: +90 216 348 02 93.
E-mail address: omer.akgiray@marmara.edu.tr (Ö. Akgiray).
[1] Present address: Marmara University, Faculty of Engineering, Department of Environmental Engineering, Istanbul, Turkey.

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Following the presentation by Bird, Stewart and Lightfoot [2], a friction factor f for flow through a packed bed of particles can be defined as follows:

$$F_k = fAK \tag{2}$$

where F_k = kinetic force (i.e. force due to motion of the fluid) exerted by the fluid on the solids, A = a characteristic area, K = a characteristic kinetic energy per unit volume, and f = friction factor. It may be noted that f is dimensionless. Choosing A to be the total external surface area of the particles that make up the packed bed and noting that the specific surface (particle surface area per unit particle volume) is given by $6/d_p$ for spheres, one can write:

$$A = \frac{6}{d_p}(1-\varepsilon)A_x L. \tag{3}$$

Here A_x represents the cross-sectional area of the empty bed. Using the Dupuit assumption, i.e. interstitial average velocity in the direction of the column axis is given by $V_\varepsilon = V/\varepsilon$, the characteristic kinetic energy is written as

$$K = \frac{1}{2}\rho\left(\frac{V}{\varepsilon}\right)^2. \tag{4}$$

A force balance on the fluid yields:

$$F_k = (-\Delta P)A_x\varepsilon. \tag{5}$$

Combining (Eqs. (2–5)) then gives:

$$f = \frac{1}{3}\frac{(-\Delta P)}{L}\frac{d_p}{\rho V^2}\frac{\varepsilon^3}{(1-\varepsilon)}. \tag{6}$$

This expression can be taken as the definition of the friction factor f. The numerical factor 1/3 is not widely used, however, and the commonly used friction factor definitions are presented below.

Ergun [1] defined the Reynolds number Re and a friction factor f_k as follows:

$$Re = \frac{d_p\rho V}{\mu} \tag{7}$$

$$f_k = \frac{-\Delta P d_p}{L\rho V^2}\frac{\varepsilon^3}{(1-\varepsilon)}. \tag{8}$$

It may be noted that $f_k = 3f$, where f is given by (Eq. (6)). It may also be useful here to note that the friction factor employed by Blake [3] and Richardson, Harker and Backhurst [4] is equal to $f_k/6 = f/2$. With the above definitions of f_k and Re, the Ergun equation (Eq. (1)) can be expressed in the following simpler form:

$$f_k = \frac{150(1-\varepsilon)}{\mathrm{Re}} + 1.75. \tag{9}$$

Ergun [1] defined another friction factor, denoted by f_V, which can be expressed as

$$f_v = \frac{-\Delta P d_p^2}{\mu V L}\frac{\varepsilon^3}{(1-\varepsilon)^2} = f_k\frac{\mathrm{Re}}{1-\varepsilon}. \tag{10}$$

A "modified particle friction factor" f_P is sometimes employed (Montillet, Akkari and Comiti [5]):

$$f_p = \frac{-\Delta P d_p}{\rho V^2 L} = f_k\frac{(1-\varepsilon)}{\varepsilon^3} = f_v\frac{(1-3)^2}{\varepsilon^3 Re}. \tag{11}$$

This quantity is also referred to as the "dimensionless pressure drop" (Eisfeld and Schnitzlein [6]). The Ergun equation can be expressed as follows in terms of these friction factors:

$$f_v = 150 + 1.75\frac{\mathrm{Re}}{(1-\varepsilon)} \tag{12}$$

$$f_p = \frac{150(1-\varepsilon)^2}{\mathrm{Re}\varepsilon^3} + 1.75\frac{(1-\varepsilon)}{\varepsilon^3}. \tag{13}$$

The Reynolds number Re appearing in these equations is defined by (Eq. (7)). A number of "modified Reynolds number" definitions are also used, the two most common being the following:

$$Re_m = \frac{d_p\rho V}{\mu(1-\varepsilon)} = \frac{Re}{(1-\varepsilon)} \tag{14}$$

$$Re_1 = \frac{d_p\rho V}{6\mu(1-\varepsilon)} = \frac{Re}{6(1-\varepsilon)}. \tag{15}$$

The latter definition can be traced back to Blake [3], whereas the definition in (Eq. (14)) differs from this definition by only the numerical factor 6. The notation Re_1 used in (Eq. (15)) is adopted from Richardson, Harker and Backhurst [4] who used it for packed beds and from Dharmarajah and Cleasby [7], Akgiray and Soyer [8] and Soyer and Akgiray [9] who employed it to correlate liquid–solid fluidization and sedimentation data. The pressure drop–flow rate correlations considered and tested in this work are listed in Table 1 in terms of f_P and Re defined as above. It may be noted that some of these equations can be written in more compact forms using alternative definitions of the Reynolds number and/or the friction factor. The Carman equation, for example, can be expressed in terms of f_k and Re_1 or Re_m in forms simpler than the rather bulky expression (Eq. (17)) in Table 1:

$$f_k = \frac{30}{Re_1} + 2.4Re_1^{-0.1} = \frac{180}{Re_m} + 2.87Re_m^{-0.1}. \tag{16}$$

The equations in Table 1, however, are all expressed in terms of f_P and Re so that they can be compared with each other directly and their differences as well as similarities can be observed more easily.

Carman [10], in a landmark paper, presented a review of earlier studies including those of Darcy [11], Blake [3], Kozeny [12], Burke and Plummer [13], and Fair and Hatch [14]. Considering the data collected by earlier researchers, Carman [10] presented an equation of the Forchheimer type (Eq. (17) in Table 1) and stated that this equation fitted the data best.

Rose [15] carried out a comprehensive analysis of data, including his own and those of other workers between 1922 and 1945. He proposed an empirical equation containing three terms (Eq. 18). It may be noted that, in this particular paper, Rose [15] did not discuss the effect of porosity on pressure drop nor did he explicitly mention the fact that his equation is applicable only at $\varepsilon = 0.4$. Furthermore, the Rose equation has been occasionally quoted (e.g. Montillet, Akkari and Comiti [5]; Özahi, Gundoğdu and Çarpinlioğlu [16]) without the porosity correction function $h(\varepsilon)$. This porosity correction function was presented in graphical form in another paper by Rose [17] and then given again in graphical form by Rose and Rizk [18] by extending it to higher porosities. Since the $h(\varepsilon)$ curve given by Rose and Rizk [18] is very inconvenient to use when a large number of calculations are needed (as is the case in this work), the following curve-fitting polynomial has been developed here to be used (within the range $0.32 < \varepsilon < 0.90$) in conjunction with the Rose equation:

$$h(\varepsilon) = 54.3218\varepsilon^4 - 156.3496\varepsilon^3 + 169.7978\varepsilon^2 - 83.0717\varepsilon + 15.6676. \tag{19}$$

It may be prudent here to emphasize that this rather bulky equation is intended to be neither a new correlation nor a suitable expression for