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Numerical investigation on angle of repose and force network from granular pile in variable gravitational environments

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ABSTRACT

This paper presents a numerical investigation on the relationship between the angle of repose and gravity by a three dimensional discrete element method. Taking particle shape, particle size and surface roughness of particle into account, the forming of granular pile was simulated under four different gravitational conditions: the Moon gravity, Martian gravity, Earth gravity and super-gravity. The angle of repose at macro level and the contact forces between particles of the pile were analyzed. Results show that the contact forces are lognormal distributed with about 65% smaller than the mean, and the ratio of tangential to normal force is uniformly distributed except for a small peak where the friction force is fully mobilized. The probability distribution of contact forces normalized by particle's weight under all gravity conditions shows to be in mutual coincidence, and the distribution of the ratio of tangential to normal force is independent of gravity. Although the microstructure of pile varies randomly, the angle of repose will not be affected by the magnitude of gravity.

1. Introduction

Surfaces of all Earth-like planets are covered with a layer of regolith which is composed of loose granular materials derived from meteoritic impacts and the subsequent production and re-accumulation of fine particles on the surface [1]. The angle of repose of granular materials is always of interest to the planetary science community. These measures are helpful in characterization and investigation of amount of planetary surface processes including the formation of sand dunes, impact craters, scree slopes, and pyroclastic cones [2,3]. And it is also useful to investigate the angle of repose for humanity when planning in situ exploration which interacts with loose surface materials [4].

The angle of repose, as a basic property of granular materials, is the maximum slope angle where the material is at rest. Above the slope angle, the material starts to flow; below the angle, the material will stop moving and becomes stable. It commonly varies from 25° for smooth spherical particles to 45° for rough angular particles in nature [5–9]. Due to the various gravity levels on the surfaces of planets, the forming processes of angle of repose might be different from each other. Though it has been assumed in planetary morphology that the angle of repose of granular material is independent of gravity [10,11], some experiments recorded in literature have contradicting results varying from opposite effects to no effect at all. For instance, Kleinhans et al. [12] conducted experiments with lower gravity using parabolic flights. They measured a number of different granular materials and determined that the angle of repose increased by roughly 5° for approximately lunar gravity. However, a study by Nakashima et al. [13], which measured the angle of repose of sands also during parabolic flights, showed that gravity had almost no effect on the angle of repose. Another study by Hofmeister et al. [14] which measured the angle of repose using the combination of a drop tower and a centrifuge, showed that the angle of repose increased when gravity decreased. Recently, the avalanche slope angles of active sand dunes on Mars were measured by Atwood-Stone and McEwen [15] in a High Resolution Imaging Experiment using high-resolution images, suggesting that these angles did not significantly vary with decreasing gravity.

Another approach to investigating this problem is through computation numerical simulation such as discrete element method (DEM), which is a powerful numerical method for analyzing microscopic behaviors of discontinuous media [16]. This has been done several times, including fairly recent studies of Ji and Shen [17] and Nakashima et al. [13], which used two-dimensional DEM to study the effect of gravity. Both of these studies determined that the angle of repose is basically independent of the magnitude of gravity, at least within the studied range of 1/6 and 1 g.

There is still not fully accepted explanation for the contradictions between research conclusions in the literatures. As Kleinhans et al. [12] had suggested, getting more microscopic information of granular system might contribute to understanding the relationship between the angle of repose and gravity. However, for the discontinuous and random

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characters of a system composed of granular materials, it is difficult to obtain some information experimentally, even by photo-elastic method, at an individual particle level such as force network which are considered as important factors related to the mechanical equilibrium of granular system [18–21].

In this work, a three-dimensional DEM was used to investigate the relationship between the angle of repose and gravity. Taking physical properties of particle into account, the accumulation process of particles had been simulated under various gravity conditions. After the formation of the granular pile, both the angle of repose at macro level and the contact forces between particles of the pile were quantitatively investigated. Compared with previous studies, this paper provides more detail information at granular level for a better understanding of the relationship between the angle of repose and gravity.

2. Numerical model and simulation conditions

Modeling of granular flows with DEM involves following the trajectories, velocities, positions of all the particles and predicting the interactions between particles by solving Newton's equation of motion for each of them. In this study, the tree-dimensional DEM with the soft contact approach, which is suitable for simulating a dense particle system, will be adopted to simulate the forming process of granular piles. Detailed principles of the DEM which were described in previous articles [22,23] will not be presented here. Simulations in this study were performed using EDEM, the engineering simulation platform developed by DEM-Solutions Company, and then particle-scale data such as contact forces would be collected and reprocessed using Distribution Fitting Toolbox provided by MATLAB. However, several key factors of modeling need to be illuminated below, such as contact force model, representation of particle shape, time step, particle generation and input parameters.

Spherical element was used to represent the spherical particle. As shown in Fig. 1, the total force acting on each particle with radius R includes gravity $m\boldsymbol{g}$, contact force $\boldsymbol{F}$ in normal and tangential directions ($\boldsymbol{F}_{\mathrm{n}}$ and $\boldsymbol{F}_{\mathrm{t}}$), accounting for the particle–particle and particle–boundary interactions. To achieve computational accuracy without increasing too amounts of calculation, the simplified Hertz–Mindlin model (Hertz–Mindlin no-slip model), which could make a good representation of the particle behavior, velocity, and contact force during collisions [24–26], was employed to figuring the contact force due to the plastic and elastic deformation in this paper. It should be noted that, for noncohesive granular particles are found in many contexts and dry flows may have occurred on Earth, Mars and some other terrestrial planets [1,12], the viscous forces such as liquid force were not considered here. Then the translational and rotational motions of particle i in the system can be described by the Newton's equation of motion:

$$m_i\frac{d^2r_i}{dt^2} = m_i\boldsymbol{g} + \sum_j\left(\boldsymbol{F}_{\mathrm{n},ij} + \boldsymbol{F}_{\mathrm{t},ij}\right) \quad (1)$$

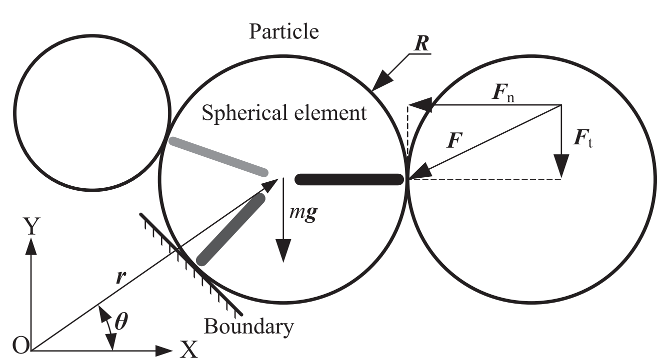

Fig. 1. Sketch of the spherical particle in DEM model.

$$I_i\frac{d^2\boldsymbol{\theta}_i}{dt^2} = -\mu_{\mathrm{r},ij}F_{ij}R_i\hat{\boldsymbol{w}}_i + \sum_j \boldsymbol{R}_i\times\boldsymbol{F}_{\mathrm{t},ij} \quad (2)$$

where $\boldsymbol{r}_i$, $\boldsymbol{\theta}_i$, $\boldsymbol{m}_i$ and $\boldsymbol{I}_i$ are the position vector, angular displacement, mass and moment of inertia of particle i, respectively; $\hat{\boldsymbol{w}}_i$ is the unit angular velocity of particle i ($\hat{\boldsymbol{w}}_i = \frac{\dot{\boldsymbol{\theta}}_i}{|\dot{\boldsymbol{\theta}}_i|}$); and $\mu_{\mathrm{r},\,ij}$ is the rolling friction coefficient between particles i and j. Using the Hertz–Mindlin no-slip model [24], the normal and tangential contact forces between particles i and j, $\boldsymbol{F}_{\mathrm{n},ij}$ and $\boldsymbol{F}_{\mathrm{t},ij}$, can be described respectively as:

$$\boldsymbol{F}_{\mathrm{n},ij} = \frac{4}{3}E^*R^{*\frac{1}{2}}\delta_n^{\frac{3}{2}} - \sqrt{\frac{20}{3}}\beta\left[E^*m^*(R^*\delta_n)^{\frac{1}{2}}\right]^{\frac{1}{2}}\boldsymbol{v}^{\mathrm{rel}}_{\mathrm{n},ij} \quad (3)$$

$$\boldsymbol{F}_{\mathrm{t},ij} = -\min\left\{\mu_{\mathrm{s},ij}F_{\mathrm{n},ij}, \left|8G^*\sqrt{R^*\delta_{\mathrm{n}}}\delta_{\mathrm{t}} + \sqrt{\frac{80}{3}}\beta\left(G^*\sqrt{R^*\delta_{\mathrm{n}}}m^*\right)^{\frac{1}{2}}\boldsymbol{v}^{\mathrm{rel}}_{\mathrm{t},ij}\right|\right\} \quad (4)$$

where the equivalent properties (radius R^*, mass m^*, Young's modulus E^* and shear modulus G^*) in terms of the properties of the particle i and j are, respectively, $R^* = \frac{R_iR_j}{R_i+R_j}$, $m^* = \frac{m_im_j}{m_i+m_j}$, $E^* = \frac{E_iE_j}{(1-\xi_j{}^2)E_i+(1-\xi_i{}^2)E_j}$, and $G^* = \frac{G_iG_j}{(1-\xi_j)G_i+(1-\xi_i)G_j}$; ξ_i and ξ_j are respectively the Poisson's ratio of particles i and j; β is the transform coefficient defined as $\beta = \frac{\ln e_{ij}}{(\ln^2 e_{ij}+\pi^2)^{0.5}}$; e_{ij} represents the restitution coefficient between particles i and j. $\boldsymbol{v}^{\mathrm{rel}}_{\mathrm{n},ij}$ and $\boldsymbol{v}^{\mathrm{rel}}_{\mathrm{t},ij}$ stand for, respectively, the normal and tangential relative velocities between particles i and j when a collision occurs (where: $\boldsymbol{n} = \frac{\boldsymbol{r}_i-\boldsymbol{r}_j}{|\boldsymbol{r}_i-\boldsymbol{r}_j|}$, $\boldsymbol{v}^{\mathrm{rel}}_{\mathrm{n},ij} = (\dot{\boldsymbol{r}}_i - \dot{\boldsymbol{r}}_j)\cdot\boldsymbol{n}$, and $\boldsymbol{v}^{\mathrm{rel}}_{\mathrm{t},ij} = \left(\dot{\boldsymbol{r}}_i - \dot{\boldsymbol{r}}_j + \boldsymbol{R}_i\dot{\boldsymbol{\theta}}_i + R_j\dot{\boldsymbol{\theta}}_j\right)\times\boldsymbol{n}$). Then $\delta_{\mathrm{n},ij}$ and $\delta_{\mathrm{t},ij}$ signify, respectively, the normal and tangential relative displacements between particle i and particle j and can be obtained through the integration of $\boldsymbol{v}^{\mathrm{rel}}_{\mathrm{n},ij}$ and $\boldsymbol{v}^{\mathrm{rel}}_{\mathrm{t},ij}$ during the collision. Note that the tangential force is bounded by $\mu_{s,ij}\mathrm{F}_{\mathrm{n},ij}$ following the Coulomb law of friction where $\mu_{s,ij}$ is the sliding friction coefficient between particles i and j. Then the force network of the pile can be obtained by linking the contact points and the center of particle, as shown in Fig. 1, where each stick represents one connection between two particles with its thickness proportional to the magnitude of a contact force.

Besides, a kind of non-spherical particle was employed here: the paired particle. As shown in Fig. 2, the multi-element method [23,27] was used to present the shape of paired particle. It was composed of two identically sized spherical elements overlapping each other. In the multi-element method, contact detection between particles is sphere-based and any other procedures for sphere–sphere contact are fully available. Then the calculation of contact forces between paired particles can be conducted with a standard discrete element algorithm for spherical particle.

In a DEM simulation, the principle commonly used to determine a time step is that the time step for calculating incremental forces and displacements of particles must be less than Rayleigh critical time step Δt_{r}, and the Δt_{r} can be shown as:

$$\Delta t_r = \pi\left[\frac{R}{0.163\xi + 0.877}\sqrt{\frac{\rho}{G}}\right]_{\min} \quad (5)$$

where ρ, ξ, and G are respectively the density, Poisson's ratio and shear modulus of particles in the system. As O'Sullivan et al. [29] and Chung et al. [30] had reported, the reasonable time step for the simulation of dense particles' movement is varied between 20% and 80% of Rayleigh critical time step. In this study, considering both of the computation time and computation accuracy, the value of 50% of Δt_{r} was chosen to simulate the forming of the granular piles.

The granular piles under variable gravity conditions were produced by the method of a point-source [13,28], which coincides with the most