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A kinetic description of collisional frictions between particles and solid boundary in simple sheared granular flows

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ABSTRACT

In this study, the collisional friction between particles and solid boundaries, which plays an important role in bounded granular flows, was investigated based on the kinetic theory for granular flows. Two limiting states of relative motion between particles and solid walls, i.e., large friction/no sliding and small friction/all sliding, as well as the transition between these limiting states were considered. In order to determine the momentum transfer between solid walls and particles experiencing apparent shears, we used a second-order approximation solution to the Boltzmann equation to determine the velocity distribution function of particles, instead of assuming a simple delta or a Gaussian distribution of particle velocity as has been done in previous studies. The second-order velocity distribution function provides a detailed vision about the energy dissipation due to inelastic particle–wall collisions as well as particle–particle collisions, as it makes it possible for us to discuss how a shear rate affects the collisional friction in bounded granular flows. Comparisons with experimental observations were made for the large friction/no sliding limit, which shows that the predicted and observed collisional friction coefficients are in well agreement. Furthermore, the calculated collisional friction coefficient coincides with computer simulations in the transition regime. Analyses show that the collisional friction is significantly affected by the shear of bounded granular flows.

1. Introduction

In granular flows, individual particles close to bed surfaces frequently interact with the confining boundaries. The interactions that occur between particles and surfaces can be either continuous or instantaneous contacts. The interactions, although being microscopic processes, behave statistically as macroscopic frictions acting on granular flows, which play an important role in the development of bounded granular flows. Therefore, the knowledge about the friction is crucial in understanding the movement of granular materials [1,2].

Different from the friction between two solid bodies, frictions stemming from either continuous contacts or instantaneous collisions between particulate materials and solid walls are not fully understood. Especially, the collisional friction, which involves complicated energy transfer and dissipation due to particle–particle and particle–wall interactions, is still an open issue requiring further studies. Previous studies on the collisional friction were mainly experimental studies focused on the friction due to relative motion between particles [3–5]. The experiments showed that the collisional friction is affected by various factors, including the shear rate of granular flows, the inelasticity of collisions, and the concentration of solids [3–8]. The most well-known experimental study is that of Bagnold [3], who conducted a series of experiments using a device with an inner stationary drum and an outer concentric rotating drum which permit measuring pressure and shear stresses acting on grains under uniform shear. Analyses show that, the collisional friction coefficient, as a key parameter which measures the intensity of the interaction between particles from a macroscopic viewpoint, appears to approach a constant in the grain-inertia regime for high-speed flows, while increases progressively through the transition regime until it reaches another constant in the viscous regime for low-speed flows. These two features have also been observed in other studies [9,10]. Savage and Sayed [4] conducted a series of experiments to investigate the stresses generated by coarse granular materials at high concentration and high shearing speed in an annular shear cell. Spherical polystyrene, glass beads, and crushed walnut of different mean diameters were used in the experiments. The experiments showed that the friction between the granular materials and the walls is influenced by concentration, slip velocity of the wall, and the fluctuation energy of particles. Most importantly, they found that the friction coefficient is highly affected by the shear rate.

Different from Bagnold and Savage and Sayed, who used annular shear cells, Pouliquen [5] carried out experimental observations on the friction between particles and an inclined rough plane. Based on the experiments, an empirical friction relation to describe the shear force was proposed as a function of mean velocity and the thickness of the granular layer [7]. The empirical relation gives the friction coefficient

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at the interface between a flowing particle layer and a rough surface along which particles flow. Nonetheless, the relation obtained by Pouliquen [5] cannot be regarded as a constitutive law since it applies only at the base of the flowing granular layer [11]. Considering the research by GDR MiDi [12], Jop et al. [11] modified the relation obtained by Pouliquen [5] by introducing the inertial number, which is a parameter to measure the relative importance of the shear rate, mean pressure, and the material properties of particles. Comparisons with experimental data indicate that the modified friction law was compatible in both plane shear and inclined-plane configurations [13]. Further study by Cassar et al. [8] illustrated that the modified friction law even holds for submarine flow. However, it should be noted that these empirical relations cannot describe the collisional interaction between particles and walls in rapid flows, because collisions between particles and walls generate rate-dependent wall friction in this case.

In addition to the experimental studies, several efforts have been devoted to theoretical descriptions of the mechanisms underlying the friction between flowing granular materials and solid walls (Jenkins and Richman [14], Lun [15], Lun and Savage [16,17], and Jenkins [18]). Among these studies, the theory of Jenkins brought about a new view on the analytical formulation of the collisional friction between particles and boundaries. Jenkins differentiates the states of relative motion between particles and a wall over which the particles moved as: small friction/large sliding, small friction/all sliding, large friction/small sliding, large friction/large sliding, and large friction/all sliding. For the cases of particles undergoing random motion, an analytical expression for the collisional friction coefficient was derived in two limiting cases (i.e., large friction/no sliding and small friction/al sliding). The theory of Jenkins [18] focused on the two extremes so that how the friction varies from one extreme to the other is unknown. Schneiderbauer et al. [19] proposed an analytical relation recently, which bridges the gap between the non-sliding case and sliding case for the collisional friction coefficient following Jenkins [18] and Louge [20]. It can be found that, in the studies of Jenkins [18] and Schneiderbauer et al. [19], a delta or a Gaussian velocity distribution was assumed in the derivation of an ensemble-averaged collisional impulses generated by the particles on the wall. This assumption implies that the flow is considered to be uniform in the near-wall regime or, more precisely, that there is no shear. However, the experiments of Savage and Sayed [4] and Hanes and Inman [6] confirmed the importance of shear on the collisional friction, as implies that their theories are insufficient when applied to granular flows with high shear rates near walls.

In this study, we aimed to extend the research of Jenkins [18] and Schneiderbauer et al. [19] by addressing the effect of shear on the collisional friction. For this reason, the velocity distribution function of particles involved in the ensemble-averaging of momentum transfer between particles and walls is determined by a second-order approximate solution to the Boltzmann equation (BE), instead of a simple delta or a Gaussian distribution. Because the second-order approximate solution to the BE includes the effect of velocity gradient of granular flows, this improvement provides a way to take into account the energy dissipation due to both inelastic particle–particle and particle–wall interactions under shearing, making it possible for us to develop a shear rate dependent relation for the collisional friction coefficient.

2. Formulation of collisional friction coefficient

For granular flows, the collisional friction on a plane wall is attributed to the collisions between particles and the wall. The collisional friction coefficient is defined as follows [18]:

$$\mu = \frac{S}{N} \tag{1}$$

where μ is the collisional friction coefficient; S and N are the shear and normal stress exerted on particles by the wall, respectively. According to different states of relative motion between particles and a wall, Jenkins [18] classified the particle motions into five special states: ①small friction/large sliding, ②small friction/all sliding, ③large friction/small sliding, ④large friction/large sliding, and ⑤large friction/all sliding. Among these states, two extreme ones, i.e., large friction/no sliding and small friction/all sliding, are of particular interest in granular flows, because they are the two basic states commonly observed for rough and smooth surfaces, respectively. For this reason, in the following subsections, the friction between particles and the wall in these two limiting states and their transition are discussed in details based on the studies by Jenkins [18] and Schneiderbauer et al. [19].

2.1. Two limiting cases

In the cases of large friction/no sliding and small friction/all sliding, the corresponding impulses on the wall generated by particle collisions can be written as [18]

$$\boldsymbol{J}^{(1)} = -m(1+e)(\boldsymbol{n}\cdot\boldsymbol{g})\boldsymbol{n} - (2m/7)(1+\beta_0)[\boldsymbol{g}-(\boldsymbol{n}\cdot\boldsymbol{g})\boldsymbol{n}] \tag{2}$$

and

$$\boldsymbol{J}^{(2)} = -m(1+e)(\boldsymbol{n}\cdot\boldsymbol{g})\boldsymbol{n} + \mu_0 m(1+e)\cot\gamma[\boldsymbol{g}-(\boldsymbol{n}\cdot\boldsymbol{g})\boldsymbol{n}] \tag{3}$$

where m is the particle mass; e and β_0 are the normal and tangential coefficients of restitution for collisions between particles and walls, respectively; $\boldsymbol{n}$ is the unit normal pointing into the flow; μ_0 is the coefficient of static friction; and γ is the angle between $\boldsymbol{g}$ and $\boldsymbol{n}$. $\boldsymbol{g}$ is the velocity at the point of contact and is given as

$$\boldsymbol{g} = \boldsymbol{c} + (\sigma/2)\boldsymbol{n}\times\boldsymbol{\omega} \tag{4}$$

while corresponding mean velocity at the contact point relative to the wall is denoted $\boldsymbol{g}_0$ and is given as

$$\boldsymbol{g}_0 = \boldsymbol{u}_0 + (\sigma/2)\boldsymbol{n}\times\boldsymbol{w} \tag{5}$$

where $\boldsymbol{c}$ is the instantaneous velocity; $\boldsymbol{u}_0$ is the average velocity of the flow at the wall that satisfies $\boldsymbol{u}_0\cdot\boldsymbol{n}=0$ because of the impenetrable wall; and $\boldsymbol{\omega}$ and $\boldsymbol{w}=\langle\boldsymbol{\omega}\rangle$ are the instantaneous and mean angular velocity of particles, respectively. Consequently, the ensemble-averaged rates of change in the momentum of a particle corresponding to Eqs. (2) and (3) are, respectively,

$$\mathbf{M}^{(1)} = \Im\left(\mathbf{J}^{(1)}\right) = N\mathbf{n} + S\mathbf{i} \tag{6}$$

and

$$\mathbf{M}^{(2)} = \Im\left(\mathbf{J}^{(2)}\right) = N\mathbf{n} + S\mathbf{i} \tag{7}$$

where $\Im$ is an integration operator given as

$$\Im(\mathbf{J}) = -\chi\iint \mathbf{J} f(\mathbf{c},\boldsymbol{\omega})(\mathbf{c}\cdot\mathbf{n})d\mathbf{c}d\boldsymbol{\omega} \tag{8}$$

in which $f(\boldsymbol{c},\boldsymbol{\omega})$ is the velocity distribution function of the particles; and the variable χ, which is a function of the solid concentration, addresses the effect of the surface on the particle spatial distribution. Apparently, the use of Eq. (8) requires knowledge of the distributions of instantaneous particle velocities and spins. However, information on the particle angular velocity is complicated and a theoretical integration for Eq. (8) is thus nearly impossible; therefore to continue the study, assumptions about the particle spin $\boldsymbol{\omega}$ should be made. In the study of Jenkins [18], two major assumptions concerning particle spin are made. The first assumption is that the mean angular velocity of spheres at the wall equals the