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A fractal model of contact force distribution and the unified coordination distribution for crushable granular materials under confined compression

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ABSTRACT

The contact forces and coordination number distributions for granular materials under breakage during one-dimensional compression are investigated using the discrete element method (DEM). The fractal distribution of contact forces within the granular packing is the main factor that induces the progressive development of the particle fractal size distribution. A simple method is developed in the current study to reveal the fractal distribution of contact forces in comminuted granular materials. It can also be used to evaluate the structural feature of granular materials. Another interesting finding is that the coordination number distribution for each particle displays a unique distribution within the fractal size distribution of granular assemblages at the ultimate state. Furthermore, the numerical results also demonstrate that the Weibull modulus, inter-particle frictional coefficient and permitted tensile stress of the initial largest particle can affect the evolution of the contact force distribution and coordination number distribution.

1. Introduction

Granular materials are applied in many fields, such as pharmaceuticals, agriculture, mining industry, construction, and so on. They are an unusual medium that cannot be directly categorized as solid, liquid or gas [1]. Particles in the granular system transfers force via the inter-particle contacts to resist external stresses; hence, the inter-particle force networks have striking features to determine the mechanical properties of the granular mass. However, even a homogeneous granular assemblage under uniform loading shows a complex and heterogonous distribution. These non-uniform force chains and particle spatial distributions have been captured in photoelastic experimental studies [2–4] through stress-induced birefringence. Hence, the probability density function (PDF) is normally considered to suitably characterize the contact force distribution, and the target is to construct a quantitative physical model for it. Coppersmith et al. [5] and Liu et al. [3] proposed the original "q model" with an exponential decay for the strong forces (greater than the average force) and a power law distribution for the weak forces (less than the average force). Afterwards, experimental studies have observed that similar results are consistent with the q-model [6]. Using numerical simulated approaches, Radjai et al. [7] investigated the contact force distribution within a two-dimensional confined packing of circular rigid disks and presented a different physical model. This model also shows that the probability of the contact normal force for the weak force follows a power law, whereas the strong-force distribution exponentially decays. In addition, the power exponent of the weak force can be negative, which is different from the q-model [8]. For the three-dimensional spherical granular assemblages, the trend of the contact force distribution has also been captured in many investigations with a similar feature as the two-dimensional condition [9–14]. These physical models can moderately predict the contact force distribution within jammed static granular assemblies under statically confined states [15]. However, for highly compressed granular assemblages, many investigations have found that the strong-force decay curve is not similar to the "exponential asymptote" relation. For example, a Gaussian distribution [16,17] and a steeper downward relation than the exponential asymptote relation [18,19] have been reported. Recently, Ben-Nun et al. [20] considered the particle-crushing effect for the polydispersity brittle granular materials during confined comminution, where the ultimate distribution of contact force decays as a clear log-normal distribution. These studies demonstrate that the contact force distribution in granular media is complex and inconsistent among different studies.

In recent years, some advanced high-resolution 3D X-ray diffraction and micro-tomography techniques have been considered to characterize particulate assemblies [21–23]. Some researchers have attempted to represent the inter-particle contact forces through the inter-particle strain fields and spatial contact distributions [24–26]. However, it remains difficult to directly obtain the accurate inter-particle contact forces within the granular system, particularly when particle crushing is considered.

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As an alternative method, the discrete element method (DEM), which was originally developed by Cundall and Strack [27], can be used to capture more information about the microscopic behaviors for a crushable granular system. A systematic review of the DEM applications and findings is presented by Zhu et al. [28] and will not be repeated here. The DEM is a more robust tool to capture the contact intensity and visualize the contact network for an individual particle [11–14, 29–35], particularly when the fracture process is considered [20, 36–39]. In addition, particle crushing makes a more complicated and heterogeneous contact system than a simple system under loading. For example, the topologies of colored grains and force networks in Fig. 1 illustrate the response to effective crushing during confined uniaxial compression in our study using the DEM. Over the past two decades, DEM has been used to investigate crushable particles through two alternative approaches: replacing the breaking particles with new smaller fragments or using a bonded agglomerate [15,20,37–50]. Different numerical models with their respective advantages and limitations are considered in terms of time efficiency, physical conservation and computing accuracy. In the current manuscript, we directly adopt a model that was systematically presented by Ben-Nun and Einav [42]. This model follows a physical conservation law to replace the pre-crushed particles with new fragments. They noted that the ultimate fractal dimension was independent of the failure criteria (tensile or shear), initial packing porosity and initial particle size distribution (PSD) using the DEM. Later, they found a new force attractor for the brittle sand during confined comminution [20]. Then, they continued to capture a stable 4-cycle loop during the comminution process through the energy scope [39]. Although some recent investigations on the contact force networks in crushable granular systems were developed, many development mechanisms of fractal fragments in brittle granular materials remain unknown. Additionally, the characteristics of the fabrics of contact forces during one-dimensional compression test are beyond the scope of this paper [14,51]. This study mainly aims to identify the features of the contact force distribution in detail using a fractal model, which is developed to analyze the normal contact force, shear contact force and total contact force in the granular packing [52]. This fractal model can also be used to explain the development of the fractal size distribution. Another core aim is to find the development of coordination number for each particle during the comminution process. Extensive numerical experiments are performed to model two-dimensional crushable granular assemblages, which are subjected to confined uniaxial compression. It is found that the development of the contact force distribution (including the normal contact force, shear contact force, and total contact force) and coordination distribution are strongly affected by the Weibull modulus, inter-particle frictional coefficient and permitted tensile stress for the initial largest particle. The attractor for the distribution of contact forces gradually emerges when the granular assemblage displays a fractal PSD. Interestingly, the distribution of the coordination numbers within the ultimate fractal granular system has a unique relation that is independent of the above variables. In addition, the size ratio between the initial maximum size and the minimum crushable size can also affect the contact forces to form a fractal distribution. In the numerical analysis, a suitable size ratio gives an acceptable result with a balance between computing efficiency and accuracy. However, this ratio is essentially an artificial factor, which appears to have no physical meaning in nature but is useful in practical computation.

This paper is organized as follows. First, we introduce the numerical DEM model for this confined comminution analysis. Then, the evolution of the contact forces and coordination number during the crushing process are described by comparing different conditions. Additionally, detailed discussions on the force intensity, force probability and particle fractal size distributions are presented in this section. Finally, a conclusion about the main findings from this study is provided.

2. Brief description of the DEM model

In the current study, the linear contact model in PFC 2D [53] is adopted to evaluate the confined comminution tests of granular assemblages. The linear contact model has been considered by many previous researchers to investigate the mechanical feature of a crushable granular system under the confined compression or shear test [15,20, 39,41–43]. Fig. 2 shows the existing linear contact model for both ball-to-ball and ball-to-wall contacts. The contact forces in PFC2D can be computed as follows:

$$f_n^c = K_n U_n, \quad \Delta f_s^c = K_s U_s, \quad f_s^c + \Delta f_s^c \le u f_n^c \tag{1}$$

where f_n^c and f_s^c are the normal and shear contact forces, Δf_s^c is the shear contact force increment, U_n is the relative normal displacement, U_s is the

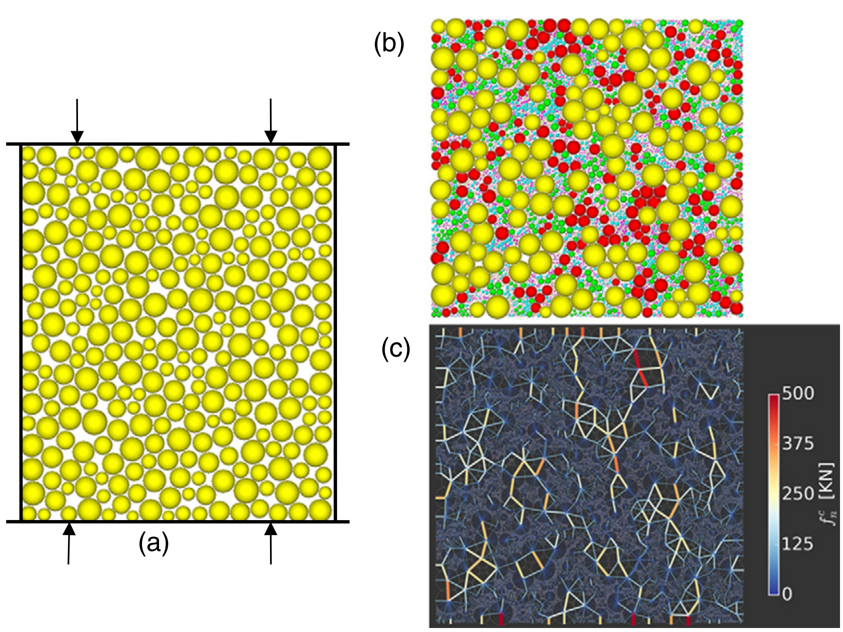

Fig. 1. Configuration of the crushable granular assemblage: (a) initial DEM sample; (b) final grains topology (vertical stress 100 MPa); (c) final contact force network (vertical stress 100 MPa).