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Break-up of bubble clusters in turbulent flow—Theory

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ABSTRACT

The behaviour of bubble clusters in turbulent conditions has been studied theoretically. The cluster behaviour was modelled based on concept drawn from the related field of bubble breakup. It was assumed that the bubbles were bridged by particles, so the cohesive strength was determined by the capillary force between the bubbles and the particles. Two different theories were investigated for the disruptive force from the turbulent liquid: the shear rate hypothesis of Camp and Stein (1943), and the turbulent fluctuation model arising from Kolmogorov's theory of isotropic turbulence (Kolmogorov, 1941). It was found that neither method is applicable in the fragmentation stage. However, in the equilibrium stage, an equation derived from Camp and Stein's theory was more realistic than that obtained using Kolmogorov's equation.

1. Introduction

In froth flotation, air bubbles are introduced to a slurry of ground particles in water where they collide with the suspended particles. The hydrophobic particles attach to the rising bubbles and are lifted up to the surface of slurry. Most flotation models are based on the collection of particles by individual bubbles. However, there are indications in the literature suggesting that hydrophobic particles are recovered with the attachment of multiple air bubbles (i.e. [14]). Indeed, the authors recently observed that bubble aggregates could easily form in a mechanical flotation cell [1]. Each bubble seemed to be held to the others by bridging particles that were attached to one or more bubbles and the shape and structure of the clusters seem to change significantly with the hydrophobicity of the particles.

Following this observation, the authors carried out the first systematic study on the clusters behaviour in a well-controlled turbulent environment [5,6]. An apparatus was used in which bubble clusters could be formed and rise into respective flow fields. The changes in the shape and size were monitored visually. Clusters properties were studied at various collector and frother concentrations, and flow conditions. It was seen that the clusters were relatively fragile, and could be disrupted by too high a level of turbulence in the cell. The experimental results indicated that a critical impeller speed exists, separating the behaviour of the bubble clusters into two stages: fragmentation and equilibrium stages. In the fragmentation stage, at low impeller speeds, the clusters were loose and filamentous, and as the energy input increased, they ruptured and re-formed [5,6]. In the second stage, above a critical impeller speed, dense clusters formed whose size was relatively insensitive to the energy input. Apart from the maximum size of the clusters, their shape factor (SF) was also examined, which was found from the measured perimeter and the projected area of the clusters [7]. It was found that although the clusters became larger at higher collector concentrations, the shape factor declined slightly, indicating a more open structure. Although these studies give an idea about the behaviour of clusters in flotation cells, at present there is a lack of understanding of the forces that hold clusters together, and the hydrodynamic conditions that are most favourable to their creation.

A field where aggregates are formed is the flocculation process for water and wastewater treatment. In this process fine particulates are caused to flocculate by the addition of reagents with charged sites that form bridges between particles in an analogous fashion to cluster formation in flotation, where the particles form bridges between bubbles. The composition of flocs does not necessarily remain the same. A continual process of growth and destruction takes place, with the ultimate diameter occurring when the rate of formation of the flocs equals the rate of destruction. The steady-state floc size is closely related to the floc strength and empirical expressions are commonly used to predict the floc strength rather than theoretical approaches due to the complexity involved in the process. The maximum floc size in a shear flow is expressed empirically as $d_{max} = CG^{-m}$ where G is the average velocity gradient, C is the strength coefficient and m is the strength constant [2,9,13]. We used this equation to study the bubble cluster breakup [7], however the equation did not provide a satisfactory agreement with the experimental results.

This paper presents a simple theoretical analysis to give insight into the behaviour of clusters under shear flow. By drawing an analogy between bubble clusters and bubble/droplet breakup, a model based on force balance is proposed to predict the maximum stable size of a bubble cluster in turbulent conditions. The cohesive capillary force and the

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detachment forces of hydrodynamic origin are balanced in the model. An important variable in the model is the shear rate used in the calculation of the dynamic detachment force. It should be borne in mind that forces acting on bubble clusters are more complicated in nature because clusters contain both bubbles and particles and the forces within clusters are not in an ordered direction. As a starting point, a simple case, such as two bubbles joined with one particle, was considered.

2. Description of theory

When a bubble is immersed in a turbulent flow, it experiences turbulent stresses including viscous stresses and dynamic pressures, which tend to deform or break the bubbles [8]. The same type of stresses could be considered to act on a cluster of bubbles held together by bridging particles. In the case of the bubble, the force that resists breakup is due to the surface tension of the bubble. In the case of the cluster, it is the internal cohesive forces arising from the capillary force between the bubbles and the bridging particles. When the disruptive forces exceed the internal force, the bubble cluster is broken up.

By comparing the disruptive and internal cohesive forces, a model can be proposed to predict the breakage of bubble clusters. For simplicity, we only consider the case that two air bubbles are connected with each other by a hydrophobic particle to avoid the effect of structural factors on the stability of bubble clusters. We will employ the following equation to express the maximum capillary force between a particle and a bubble, from Nguyen [12] in the form:

$$F_c = 2\pi R_p \sigma \sin^2(\theta/2) \tag{1}$$

where R_p is the radius of particle, σ is the gas–liquid surface tension and θ is the contact angle (see Fig. 1).

The disruptive dynamic pressure force is equal to the dynamic pressure multiplied by the area where the pressure applies, which can be taken to be the area of cross-section of an individual bubble cluster, πR_c^2, where R_c is the radius of cluster. Thus, the dynamic pressure force can be expressed as:

$$F_d = \pi R_c^2 \rho_l \overline{u_d^2}/2 \tag{2}$$

where ρ_l is the density of fluid, and $\overline{u_d^2}$ is the mean square velocity difference over a distance equal to the diameter of the bubble cluster, d_c.

Two assumptions can be made to predict the value of $\overline{u_d^2}$. Firstly, it will be assumed that the velocity difference can be calculated by relating it to the shear rate. If a cluster is in a uniform shear field, the velocity difference at opposite ends of a cluster is $\Delta u_d = G d_c$, where d_c is the cluster diameter and G is the shear rate. The shear rate can be calculated from the equation of Camp and Stein [3] as: $G = \sqrt{\varepsilon/\nu}$, where ν is kinematic viscosity, and ε is energy dissipation per unit mass of fluid, which is given by $\varepsilon = P_0 N^3 D^5/V$, where P_0 is the impeller power number, N is the impeller speed, D is the impeller diameter, and V is the volume of the liquid in the stirred tank. By substituting all parameters, one can obtain:

$$\left(\overline{u_d^2}\right)_C = \varepsilon d_c^2/\nu \tag{3}$$

where the subscript C of $\overline{u_d^2}$ denotes that this expression uses the Camp and Stein expression.

By substituting Eq. (3) into Eq. (2), the dynamic pressure force can be modified to:

$$(F_d)_C = \pi \rho_l \varepsilon d_c^4/8\nu. \tag{4}$$

The value of $\overline{u_d^2}$ can also be obtained according to Kolmogorov's theory of local isotropy [10]. It is assumed that at sufficiently high Reynolds numbers (>10,000) the micro-scale components of the turbulent velocity fluctuations are isotropic and therefore independent of the main flow and the generating mechanism of the flow. In the case of a droplet in an isotropic turbulent flow, that is much larger than the micro-scale, the mean square of the relative flow velocity over a distance equals to the diameter of droplet d is independent of viscosity, which can be expressed as [15]:

$$\left(\overline{u_d^2}\right)_K = 2\varepsilon^{2/3} d_c^{2/3} \tag{5}$$

where the subscript K of $\overline{u_d^2}$ denotes that this expression uses Kolmogorov's theory. An analogy can be drawn with the case of a cluster, where the cluster size is much larger than the microscale, in which case d_c is the diameter of the cluster.

By substituting Eq. (5) into Eq. (2), the dynamic pressure force can be modified to:

$$(F_d)_K = \pi \rho_l \varepsilon^{2/3} d_c^{8/3}/4 \tag{6}$$

and it can be observed that the values of F_d depend in different ways on the key variables ε and d_c.

For real systems, one bubble may be connected with many particles so that many joints may be broken up at the same time. If the average number of broken contacts of clusters is N_c it would be reasonable to assume that for stability, the dynamic pressure force should be balanced by N_c times the capillary force. By balancing of Eqs. (4) and (6) with

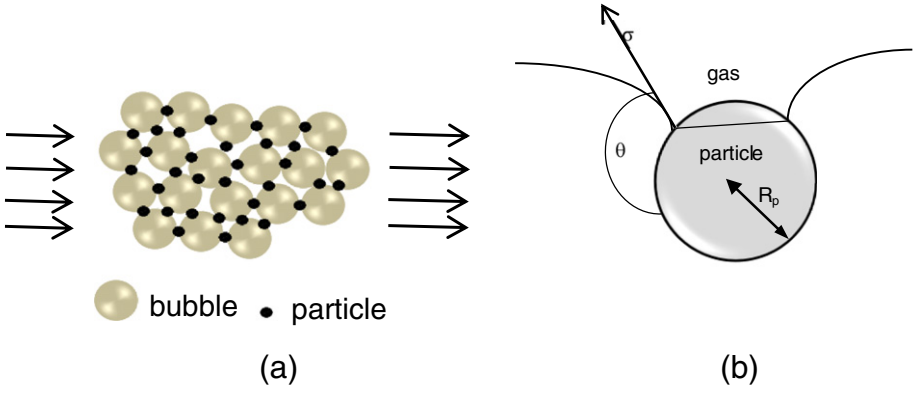

Fig. 1. Bubble cluster in turbulent flow. Dynamic pressure force due to the flow velocity difference at opposite ends of the cluster causes the cluster deformed and breakup (a). At steady-state this force is balanced by the capillary force which is function of the particle radius R_p, the gas–liquid surface tension σ and the contact angle θ (b).