Powder Technology 273 (2015) 102–110

Contents lists available at ScienceDirect

Powder Technology

journal homepage: www.elsevier.com/locate/powtec

A stress discontinuity approach to model the stress profile on a loaded conveyor belt

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ARTICLE INFO

Article history:
Received 9 September 2014
Received in revised form 17 December 2014
Accepted 20 December 2014
Available online 27 December 2014

Keywords:
Belt conveyor
Bulk material
Stress profile
Idler
Stress discontinuity

ABSTRACT

The prediction of the stress profile on a loaded conveyor belt is essential not only for the engineering design of the belt, but for the analyses of the interactions between the belt and neighbouring components like idlers. This paper presents a new analytical approach by which the stress profile on a loaded conveyor belt can be modelled. We propose a new hypothesis of the dynamic movement of bulk material during conveyance based on recent experimental discovery. We investigate the internal friction within bulk material, as well as the interface behaviour between bulk material and the belt. The presented analytical approach incorporates both the stress field analysis and the Stress Discontinuity method. The resulting Stress Discontinuity model can fully characterise active and passive stress states within bulk material. Compared with results of on-site experiments and a widely accepted previous model, it shows that a good correlation can be achieved. The Stress Discontinuity model is verified by comparison to a series of experimental data.

1. Introduction

As large-scale continuous material transport systems, belt conveyor systems are employed to convey large volume of bulk material rapidly and efficiently [1]. The determination of the stress profile on the belt induced by bulk material is essential. The stress profile is critical for the engineering design of the belt with required strength and proper tension. The estimation of the load on neighbouring components like idlers and pulleys also largely depends on the stress profile on the belt. Moreover, the stress profile plays an important role in realistic prediction of the belt wear, the rolling resistance analysis [2], and consequently the energy consumption calculation.

The analysis of the stress profile on the belt is very complicated due to cyclic active and passive stress states in bulk material [3], unascertained interface behaviour between the belt and bulk material, and the effect of the belt sag during conveyance [4]. Active and passive stress states occur simultaneously due to the opening and closing movements of the belt passing over consecutive idlers. To enable a better description, the belt is artificially divided into two Wing Belt Sections (WBSs) supported by wing rolls and one Centre Belt Section (CBS) supported by the centre roll (Fig. 1). Traditionally, Krause and Hettler [5] assumed that active and passive stress states are induced by the pivoting of the WBS about the joint between the WBS and the CBS. Based on 3D scanned geometry (Fig. 1), instead, Ilic [6] recently suggested that active and passive stress states are more likely caused by the pivoting of the WBS about the outer edge of the belt. In Fig. 1, the outer edges of the belt have small deflections between the maximum belt sag (at the middle of an idler spacing) and zero belt sag (above an idler) while the joints have relatively large deflections between the two positions.

Previous research on this topic can be categorised into theoretical research and experimental study. In theoretical research, Krause and Hettler first developed an analytical model (KH model) based on Coulomb earth pressure theory [5]. Assuming two planar failures through the joints within bulk material, one can obtain the stress profile using soil failure mechanics. However, experimental results show that the KH model largely overestimates the load on the WBS while underestimates the load on the CBS [6,7]. Wheeler [8] carried out a numerical model to calculate the pressure on the belt which included the influence of the belt sag. Plate mechanics was applied to discretize the belt. Applying the Discrete Element Method (DEM), Ilic [6] carried out a model to analyse the stress profile. Measured belt deflections were taken as inputs of the model. The results from his model indicate that the particle size largely affects the stress profile. To date, numerical solutions are still complicated and prohibitively time-consuming for applications.

The stress profile has been experimentally studied mostly through the measurement of forces on idler rolls. Different laboratory and on-site experimental apparatuses have been developed by researchers in this field, including Grabner [9], Geesmann [10], Wheeler [7] and Ilic [6]. Study from these experiments indicates the influence of bulk material properties, the belt tension, the belt characteristics as well as the idler configuration on the stress profile or the forces on idler rolls.

This paper presents a novel analytical approach to model the stress profile on the belt. Bulk material is divided into two zones, the free

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http://dx.doi.org/10.1016/j.powtec.2014.12.040
0032-5910/

Nomenclature

f_d	dynamic factor [–]
g	acceleration of gravity [m/s^2]
l	idler spacing [m]
m_{bulk}'	mass of bulk material per unit length [kg/m]
v	belt velocity [m/s]
x	distance along the wing belt section [m]
z	depth of bulk material element from surface [m]
$F_{NG,bulk,a}$, $F_{NG,bulk,p}$	normal forces on the belt in active and passive stress states [N]
$F_{NA,bulk,a}$, $F_{NA,bulk,p}$	shear forces on the belt in active and passive stress states [N]
$F_{NG,c}$	normal force on the centre roll [N]
$F_{NG,w}$	normal force on the wing roll [N]
$F_{NA,w}$	axial force on the wing roll [N]
L_1	length of bulk material along the wing belt section [m]
K_a, K_p	coefficients in active and passive stress states [–]
K_s	belt sag ratio [–]
R_1, R_2	radiuses of Mohr circles [N/m^2]
α	trough angle of the idler configuration [–]
β	conveyor surcharge angle of bulk material [–]
δ	strength mobilized on the stress discontinuity [–]
θ, θ_o	rotations of the major principal stress [–]
θ_a, θ_p	rotations of the major principal stresses in active and passive stress states [–]
ρ	density of bulk material [kg/m^3]
σ_{OC1}, σ_{OC2}	average effective stresses in active stress state [N/m^2]
$\sigma_{w,a}$, $\sigma_{w,p}$	normal stresses in active and passive stress states in zone 2 [N/m^2]
$\tau_{w,a}$	shear stress in active stress state in zone 2 [N/m^2]
φ_i	internal friction angle of bulk material [–]
φ_w	wall friction angle between the belt and bulk material [–]
Δ, Δ_1, Δ_2, Δ_3, Δ_4	angles in Mohr circle [–S]

surface zone and the interface zone. We propose a new hypothesis of the dynamic movement of bulk material in the two zones. Based on the proposed new hypothesis, we carry out the stress field analyses in the two zones according to active and passive stress states separately. The change of the major principal stress from the free surface zone to the interface zone is analysed using the Stress Discontinuity method (SD method). In addition, the effect of the belt sag is also taken into consideration by applying a dynamic factor.

The paper is organized as follows. Section 2 briefly introduces the principles of the SD method. Section 3 presents detailed analysis of the hypothetical movement of bulk material, as well as the development of the Stress Discontinuity model (SD model). Section 4 provides the verification of the SD model by on-site experiments, and some results and comparisons with the KH model are discussed. Principal findings and conclusions are presented in Section 5.

2. Principles of SD method

Sokolovski [11] set out the SD method to solve complicated civil engineering problems. The concept of the stress discontinuity in the SD method was introduced to describe the variation of the effective stress. Examples of stress discontinuities can be found in failures of soil bodies [12], the silo storage with mass flow [13], shear bands in soil–structure interfaces [14] and so on.

Fig. 2 (A) presents a stress discontinuity which separates two bodies of bulk material under different stress states. Each body consists of numerous infinitesimal bulk elements. In Fig. 2(B), Mohr circles are built to describe the stress states of bulk elements in body 1 (circle C_1) and body 2 (circle C_2) separately. For two bulk elements located at either side of the stress discontinuity, the stresses normal to the discontinuity plane must be equal to each other, while the normal stresses parallel to the plane can be different. As a result, Mohr circles of the two bulk elements intersect. The intersection [point A in Fig. 2(B)] depicts the normal stress and shear stress of the facets of both elements that are parallel to the discontinuity plane.

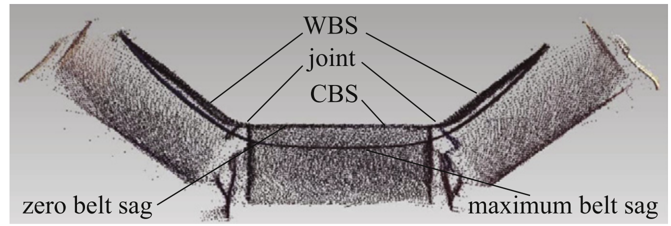

Fig. 1. Belt cross-sectional profile: 3D laser scan belt geometry [6].

With the two intersected Mohr circles, the relationship of the effective stresses across the stress discontinuity can be retrieved. In most engineering problems, the direction and magnitude of the effective stress at one side of the discontinuity are already known, while the effective stress at the other side remains unclear. Assuming that the effective stress increases from body 1 to body 2, the average effective stresses σ_{OC_1} and σ_{OC_2} [Fig. 2(B)] have the following relationship [11]:

$$\frac{\sigma_{OC_2}}{\sigma_{OC_1}} = \frac{\sin(\Delta+\delta)}{\sin(\Delta-\delta)} \tag{1}$$

where Δ is an angle in Mohr circle [Fig. 2(B)] and $\sin\Delta = \sin\delta/\sin\varphi_i$, φ_i is the internal friction angle, and δ is the angle that indicates the strength mobilized on the discontinuity and $\tan\delta = \tau/\sigma$.

Moreover, the rotation of direction of the major principal stress can also be obtained. In body 1, the plane that the major principal stress σ_1 acts on is at an angle of $(\Delta+\delta)/2$ from the stress discontinuity plane [Fig. 2(A)] while the plane for σ_2 is at an angle of $(\pi-\Delta+\delta)/2$ from the stress discontinuity in body 2 [Fig. 2(A)]. As a result, the rotation of direction of the major principal stress across the stress discontinuity is obtained:

$$\theta = \frac{\pi}{2} - \Delta \tag{2}$$

where θ is the rotation angle of the major principal stress [Fig. 2(A)].

Extra attention has to be paid to the selection from the two intersections of Mohr circles. In reality, the two intersections represent the effective stresses of the same magnitude but in different directions. This may result in different relationships of the effective stresses across the stress discontinuity. The selection of the intersection is generally determined by the stress state in body 1, the reference axes as well as the definition of the positive stress during the construction of Mohr circles.

After analysing the condition of one stress discontinuity, the variation of the effective stress across a number of stress discontinuities can be considered. For each infinitesimal stress discontinuity, successive rotation and change in the major principle stress occur. For cohesionless bulk material, the two Mohr circles are almost overlapping with each other, so the mobilized strength δ tends to be the internal friction angle φ_i. As a result, for an overall rotation of angle θ_o of direction of the principal stress, the relationship of the average effective stress across the stress discontinuities can be obtained [15]:

$$\frac{\sigma_{OC_2}}{\sigma_{OC_1}} = e^{2\theta_o \tan\varphi_i}. \tag{3}$$