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Optimization of gravity flow discharge chutes under the speed dependent resisting forces: Maximizing exit velocity

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ABSTRACT

Using the optimal control theory, the problem of finding profiles of gravity flow discharge chutes required to achieve maximum exit velocity of granular material under the speed dependent resisting forces is solved. A model of a particle moving down a curve which is treated as a unilateral constraint is used. The fast flow condition and the condition that the particle does not leave the curve are introduced as the additional inequality constraints. The influence of the initial particle speed and the power of the speed in the expression for the resisting force on the optimal chute profile are analyzed.

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1. Introduction

This paper considers the problems of the optimization of gravity flow discharge chute profiles in bulk granular materials handling installations. Fig. 1(a) displays a principle scheme of such an installation. From the bin (1), the granular material (2) moves by means of the feeder (3) into the loading chute (4). At the exit of the chute, the material is delivered to the conveyor (5) (note that some other storage device can be this component of the system). A granular material flows along a chute under the action of its own weight and that is why in the literature such chutes are called gravity flow discharge chutes. In chute profile optimization the most common optimization criteria are the minimization of the transit time of granular materials and the minimization of the losses of mechanical energy of granular materials due to the friction. The last criterion is often expressed as maximization of the exit velocity of the granular material. For the other optimization criteria for this type of installation see [1,2].

In the reference [3] it was shown that in case when a material flows through a chute in the form of fast flow, the flow of material through the chute can be modeled as a particle M moving down a curve with tangentially directed resisting forces (see Fig. 1(b)). The curve is treated as a unilateral constraint because the open chutes are considered. This means that the particle must slide along the curve like a block on an inclined plane. The shape of this curve should be such that the particle M starting from the position $M_0(x_0,y_0)$ with the initial speed V_0 reaches the position $O(0,0)$ either for the minimal time or the maximal speed (minimal losses of mechanical energy). In Fig. 1(b), y represents the vertical axis directed downwards, and x is the horizontal axis of a Cartesian coordinate system.

In the reference [4] considerations of the problem of maximum exit velocity of granular material under the speed dependent resisting forces do not take into account the fast flow condition and the condition that the particle does not leave the chute. Consequently, the results obtained in [4] refer to those values of the model's parameters that ensure the satisfaction of the previous conditions without their explicit incorporation in the equations of the problem. A similar problem, without connection with the problem of optimization of discharge chutes, was considered in [5]. The solution for the problem of maximum exit velocity under the Coulomb friction force as well as a review of literature relating to this problem is given in [6].

In this paper, using the optimal control theory [7,8], the problem of maximization of exit velocity is solved by directly introducing the fast flow condition and the condition of non-leaving the chute bottom. To the best of the authors' knowledge, solving the considered problem by using these two conditions has not been reported elsewhere before. These conditions, considering the model of a particle used, are represented by equivalent conditions that the particle tangential acceleration is larger or equal to zero and that during motion the reaction of the chute does not change the direction. The resisting force that depends on the particle velocity is considered. The numerical procedure for solving the problem is based on the shooting method [9]. The determination of optimal chute profiles is illustrated via examples. The obtained chute profiles are compared with those existing in literature.

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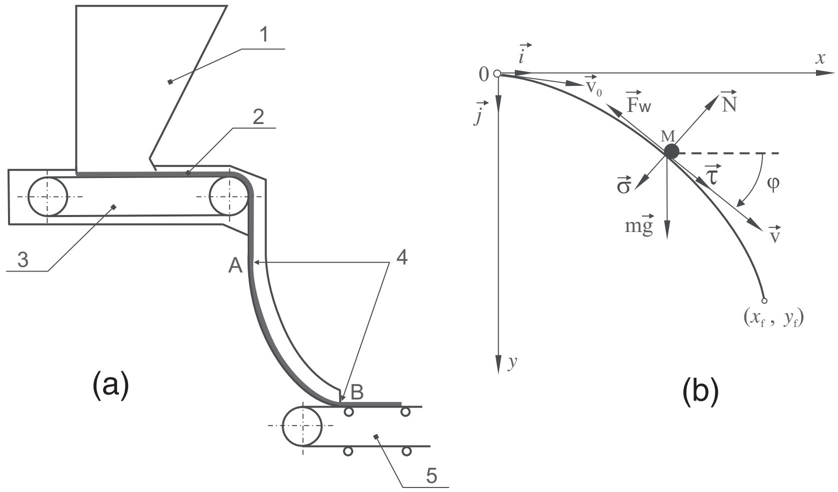

Fig. 1. (a) Gravity flow discharge chute; (b) The physical model of the chute.

2. Optimal control formulation

The differential equation of motion of the particle M shown in Fig. 1(b) reads:

$$m\vec{a} = m\vec{g} + \vec{N} + \vec{F}_W. \tag{1}$$

where $\vec{a}$ is the acceleration of the particle, $\vec{g} = g\vec{j}$, g is the acceleration of gravity, $\vec{N}$ is the normal component of the constraint reaction force, and $\vec{F}_W$ is the resisting force.

Let us introduce the unit vectors $\vec{\tau}$ and $\vec{\sigma}$ in the following way [10] (see Fig. 1(b)):

$$\vec{\tau} = (\cos\varphi)\vec{i} + (\sin\varphi)\vec{j}, \tag{2}$$

$$\vec{\sigma} = \frac{d\vec{\tau}}{d\varphi} = (-\sin\varphi)\vec{i} + (\cos\varphi)\vec{j}, \tag{3}$$

where $\vec{i}$ and $\vec{j}$ are the unit vectors of axes x and y, respectively, and $\vec{\tau}$ and φ are the unit vector and the slope angle of the tangent to the particle path, respectively. It is obvious that $\vec{\tau}\cdot\vec{\sigma} = 0$. The reason to introduce vector $\vec{\sigma}$ is that the unit vector $\vec{\sigma}$, in contrast to the principal normal unit vector of the particle path, does not change the orientation with changing of the concavity of the curve and it is constantly directed to the same side with $\vec{g}$ (see Fig. 1(b)). Now, the acceleration $\vec{a}$ can be written as (see [10])

$$\vec{a} = \dot{V}\,\vec{\tau} + V\,\dot{\varphi}\,\vec{\sigma} \tag{4}$$

where an overdot denotes the derivative with respect to time t and V represents the projection of the particle velocity on the direction $\vec{\tau}$. Also, in regard to Fig. 1(b), the following kinematics relations hold:

$$\dot{x} = V\cos\varphi, \qquad \dot{y} = V\sin\varphi. \tag{5}$$

In further considerations it is assumed that the force $\vec{F}_W$ has the following form:

$$\vec{F}_W = -mR(V)\vec{\tau} \tag{6}$$

where $R(V)$ is the resisting force per unit mass of the form

$$R(V) = \beta V^k. \tag{7}$$

In Eq. (7), β denotes the friction coefficient in dimension m^{1-k}/s^{2-k} and $k \in R$. In regard to above, projecting Eq. (1) on the directions $\vec{\tau}$ and $\vec{\sigma}$ yields

$$\dot{V} = g\sin\varphi - \beta V^k, \tag{8}$$

$$mV\dot{\varphi} = mg\cos\varphi + N_\sigma, \tag{9}$$

where $N_\sigma = \vec{N}\cdot\vec{\sigma}$. In accordance to the lumped particle model used, the condition that the particle does not leave the curve can be expressed in the form

$$N_\sigma \le 0, \tag{10}$$

and the condition that the flow of a granular material through the discharge chute is fast [3] can be expressed through the following equivalent condition imposed to the particle tangential acceleration:

$$\dot{V} \ge 0. \tag{11}$$

In order to formulate a optimal control task, let us introduce a new variable p and a control variable u as in [11]:

$$p \triangleq \tan\varphi, \quad u \triangleq \frac{dp}{dx}. \tag{12}$$

Now, in accordance to Eqs. (5), (8), and (12) and taking the quantities y, p, and V as states, the following state equations can be formed:

$$\frac{dy}{dx} = p, \tag{13}$$

$$\frac{dp}{dx} = u, \tag{14}$$

$$\frac{dV}{dx} \triangleq f_V(p, V) = \frac{gp - \beta V^k\sqrt{1+p^2}}{V}, \tag{15}$$

with the prescribed initial and terminal conditions

$$x = 0: \; y(0) = 0, \qquad V(0) = V_0; \qquad x = x_f: \; y(x_f) = y_f. \tag{16}$$

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