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A 3-parameter packing density model for angular rock aggregate particles

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ABSTRACT

The authors have in recent studies incorporated the wedging effect to develop a 3-parameter model for packing density prediction of binary and ternary mixes of spherical particles. This model has been restricted to only spherical particles because only the test results of spherical particles were used for derivation and validation. For more general applications to other types of particles, such as angular particles, the model needs to be further developed. In this study, an experimental program on the packing density of binary mixes of angular rock aggregate particles was carried out and the experimental results were used to derive the interaction functions of the three parameters (the loosening, wall and wedging effect parameters) for extending the 3-parameter model to binary mixes of angular particles. Apart from the experimental results obtained herein, the test results published by de Larrard and those obtained by the authors in an earlier study were also used to validate the extended 3-parameter model.

1. Introduction

Particulate materials are widely used in many processing industries, such as the mineral, metallurgical, pharmaceutical and food industries [1–4]. Product design and manufacturing across this wide spectrum of industries are largely a practice of particle technology, which generally requires the knowledge of particle packing. For this reason, the importance of particle packing is well known in the processing industries, but somehow not in the concrete production industry. Actually, a typical concrete mix may be regarded as a mixture of aggregate particles and cement paste. The cement paste fills the voids between the aggregate particles and binds the aggregate particles upon hardening to give strength to the concrete. Among the various ingredients, cement is the most expensive and has the highest carbon footprint [5,6]. To slash the cost and carbon footprint of concrete production, a promising way is to maximize the packing density of the aggregate so as to reduce the volume of cement paste needed to fill the voids [6]. For this task, it would be helpful to have a particle packing model for predicting the packing density of rock aggregate particles, which are usually angular in shape.

Numerous particle packing models have been developed but most of these models are limited to spherical particles. In the 1930s, Westman and Hugill [7] and Furnas [8] pioneered to derive analytical equations for predicting the packing density of binary mixes of spherical particles by considering two extreme cases: first, when the size ratio is close to zero, and second, when the size ratio is close to unity (the size ratio is the ratio of the size of smaller particles to the size of larger particles). Since then, substantial progress has been made in the packing density prediction of binary mixes of spherical particles with an intermediate size ratio, in which case, interactions between particles of different sizes occur [9–16]. Two interaction effects have been identified, namely: the loosening effect, which occurs when the larger particles are dominant and the wall effect, which occurs when the smaller particles are dominant. These two effects are taken into account using two parameters, called the loosening effect parameter and the wall effect parameter, respectively. Both these two parameters are functions of the size ratio (these functions are called interaction functions) derived by curve fitting against experimental results.

The particle packing models, which employ two parameters to account for the loosening and wall effects, may be referred to as the 2-parameter models. In most of the 2-parameter models [10,12,13,16], the relationship between the specific volume of the particle system and the volumetric fractions of the various size classes of particles (each size class is a collection of mono-sized particles) is assumed to be linear. However, it has been observed from test results [9,17,18] that the relationship between the specific volume and the volumetric fractions is in fact nonlinear. To take into account such nonlinear relationship and improve the accuracy of the 2-parameter models, de Larrard [17] incorporated a compaction index, which is dependent on the packing process, to develop a more advanced compressible packing model, whereas Kwan et al. [18] added a wedging effect parameter, which accounts for a newly identified wedging effect, to develop a more sophisticated 3-parameter model. The 3-parameter model developed

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by Kwan et al. [18] is only for binary mixes of spherical particles. Later, Wong and Kwan [19] refined this model for application to ternary mixes of spherical particles.

Most existing particle packing models are applicable only to spherical or rounded particles because they were validated by comparing with experimental results of only spherical or rounded particles. For more general applications to non-spherical particles, further development of the particle packing models to allow for the effects of particle shape is needed. Overall, the particle shape may have two effects: first, on the packing density of each mono-sized fraction of the particles, and second, on the interactions between the different mono-sized fractions of particles. The effect of particle shape on the packing density of mono-sized particles has been studied extensively using regular particles such as ellipsoids, spheroids, disks and cylinders [20,21]. Moreover, the packing density of irregular rock aggregate particles has been measured and correlated to the various shape parameters [22]. Anyway, whatever the effect of particle shape is, the packing density of mono-sized particles can be measured directly, and provided the measured packing densities of the mono-sized fractions are used in the packing density prediction, the effect of particle shape on the packing density of mono-sized particles should already be allowed for.

To characterize the size of a mono-sized fraction of non-spherical particles, Yu and Standish [23] and Yu et al. [24] introduced the concept of equivalent packing diameter and employed this concept to extend their 2-parameter model for application to non-spherical particles. Meanwhile, Goltermann et al. [25] modified the packing models of Aïm and Goff [10] and Toufar et al. [11] for application to rock aggregates and introduced the concept of position parameter for characterizing the size of aggregate particles having a certain particle size distribution ranging over several sieve sizes. On the other hand, for aggregate particles passing a certain sieve (the upper sieve) but retained on the next smaller sieve (the lower sieve), of which the size range is relatively narrow, de Larrard [17] assigned the geometric mean of the upper and lower sieve sizes to be the characteristic size.

Herein, the 3-parameter model [18,19] is extended for application to binary mixes of angular rock aggregate particles. The measured packing densities of the mono-sized fractions are used in the packing density prediction and thus the effect of particle shape on the packing density of each mono-sized fraction should have been allowed for. An experimental program has been carried out to provide data for deriving the interaction functions. Two sets of interaction functions are derived, one for the uncompacted condition and the other for the compacted condition. Finally, the extended 3-parameter model is compared to the test results obtained in this study and the test results by de Larrard [17] and by Fung and Kwan [26] to verify the applicability and accuracy of the model.

2. The 3-parameter model for spherical particles

The 3-parameter model was originally developed by Kwan et al. [18] for binary mixes of spherical particles. It requires pre-determination of the optimum volumetric fractions yielding the maximum packing density before applying the equations contained therein to predict the packing density. For easier application and dealing with ternary mixes, the 3-parameter model has been refined by Wong and Kwan [19] in two ways. First, the equations are modified so that they are not dependent on the optimum volumetric fractions and pre-determination of the optimum volumetric fractions is not needed any more. Second, the equations are expanded to cater for the packing density prediction of ternary mixes. The refined 3-parameter model by Wong and Kwan [19] is adopted herein as the basic framework for extension to apply to angular particles. For easy reference, an outline of this 3-parameter model is presented in the following.

Consider a binary mix composed of two size classes of mono-sized spherical particles: size class i and size class j, in order of increasing particle size. Let the diameters of particles of size class i and size class j be d_i and d_j, respectively (note that $d_i \le d_j$), the volumetric fractions of size class i and size class j be r_i and r_j, respectively (note that $r_i + r_j = 1$), and the packing densities of size class i and size class j be ϕ_i and ϕ_j, respectively. The packing density of the binary mix when size class i is dominant (denoted by $\phi_i{}^*$) and the packing density of the binary mix when size class j is dominant (denoted by $\phi_j{}^*$) are given respectively by:

$$\frac{1}{\phi_{i*}} = \left(\frac{r_i}{\phi_i} + \frac{r_j}{\phi_j}\right) - \left(1 - b_{ij}\right)\left(1 - \phi_j\right) \cdot \frac{r_j}{\phi_j} \cdot \left[1 - c_{ij}\left(2.6^{r_j} - 1\right)\right] \quad (1)$$

$$\frac{1}{\phi_{j*}} = \left(\frac{r_i}{\phi_i} + \frac{r_j}{\phi_j}\right) - \left(1 - a_{ij}\right) \cdot \frac{r_i}{\phi_i} \cdot \left[1 - c_{ij}\left(3.8^{r_i} - 1\right)\right] \quad (2)$$

in which a_{ij}, b_{ij} and c_{ij} are the loosening effect parameter, wall effect parameter and wedging effect parameter, respectively, and are given as functions of the size ratio. The final packing density of the binary mix of particles can be determined simply as min.{$\phi_i{}^*$, $\phi_j{}^*$}.

The loosening effect occurs in the case when the larger particles are dominant due to the smaller particles squeezing into the voids between the larger particles causing loosening of the packing of the larger particles. The wall effect occurs in the case when the smaller particles are dominant due to the boundaries of the larger particles acting like walls causing disruption of the regular packing of the smaller particles. However, the wedging effect takes place in both cases. In the case when the larger particles are dominant, the wedging effect occurs when some isolated smaller particles are entrapped in the gaps between the larger particles instead of filling into the voids thereby wedging the larger particles apart. In the case when the smaller particles are dominant, the wedging effect takes place when some gaps between the larger particles are too narrow to accommodate even one complete layer of smaller particles leading to the presence of only isolated smaller particles at the gaps wedging the larger particles apart. Fig. 1 demonstrates the loosening effect, wall effect and wedging effect in binary mixes of particles.

It should be noted that if the values of c_{ij} in Eqs. (1) and (2) are set to zero, Eqs. (1) and (2) would revert back to the packing density equations in the conventional 2-parameter models. With the additional terms accounting for the wedging effect, the relationship between the specific volume and the volumetric fractions is no longer linear because the wedging effect varies non-linearly with volumetric fractions. For spherical particles, Kwan et al. [18] and Wong and Kwan [19] have derived the following interaction functions for the evaluation of the loosening effect, wall effect and wedging effect parameters:

$$a = 1 - (1 - s)^{3.3} - 2.6 \cdot s \cdot (1 - s)^{3.6} \quad (3a)$$

$$b = 1 - (1 - s)^{1.9} - 2.0 \cdot s \cdot (1 - s)^{6.0} \quad (3b)$$

$$c = 0.322 \cdot \tanh(11.9 \cdot s) \quad (3c)$$

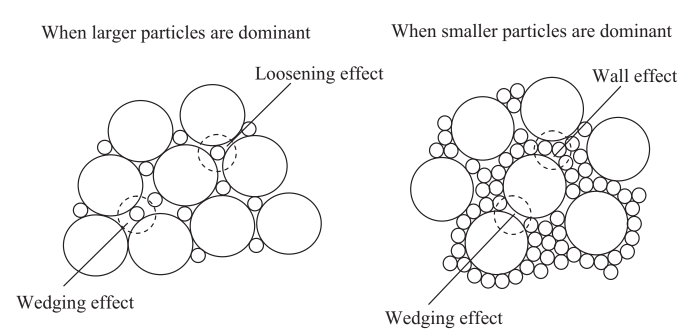

Fig. 1. A schematic diagram showing the loosening, wall and wedging effects.