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Motion analysis of a spherical solid particle in plane Couette Newtonian fluid flow

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ABSTRACT

In this article, the motion of a spherical particle in a plane Couette Newtonian fluid flow is studied. The governing equations of a spherical solid particle's motion in the plane Couette fluid flow are investigated using the Differential Transformation Method (DTM) and a Padé approximant which are an analytical solution technique. For validation of the analytical solution, the governing equation is solved numerically. The horizontal and vertical velocities of spherical solid particle are shown for different fluids and values of the embedding parameters. The DTM–Padé results indicate that the horizontal and vertical velocities of spherical solid particle in water fluid are higher than the glycerin and ethylene-glycol fluids. Also the horizontal and vertical velocities increase with an increase in the particle radius. Comparison of the results (DTM and numerical) was shown that the analytical method and numerical data are in a good agreement with each other.

1. Introduction

The fluid–solid flow particles are occurring in various industrial and natural applications. Knowledge of the behavior of solid particles allows the treatment and improvement of several industrial processes in the field of process engineering, e.g. drying, agitation, mixing and separation as well as aerospace and, etc. The transport of arbitrarily shaped particles is of great importance also in several biomedical applications: particles of various shapes, e.g. spheres, disks and rods have been developed for controlling and improving the systemic administration of therapeutic and contrast agents. Owing to the importance of the aforesaid applications, considerable attention has been devoted to the study of the accelerated motion of a sphere in a fluid, and an excellent account of the theoretical developments in this area has been given by R. Clift et al. [1] for Newtonian fluids. More recently, several studies were performed on spherical and non-spherical solid particles. M. Hatami et al. [2] applied the Multi-step Differential Transformation Method (Ms-DTM) on particle's motion in Couette fluid flow with considering the rotation and shear effects on lift force and neglecting gravity. M. Hatami and D.D. Ganji [3] solved the equation of a particle's motion on a rotating parabolic surface with the Multi-step Differential Transformation Method (Ms-DTM) and achieved comparable results to the numerical ones. The motion of a particle in a fluid forced vortex was studied by M. Hatami and D.D. Ganji [4]. They applied the Differential Transformation Method (DTM) and Differential Quadrature Method (DQM) to solve governing equations. The unsteady settling behavior of solid spherical particles falling in the water as a Newtonian fluid was investigated by R. Nouri et al. [5]. They applied three different analytical methods to analyze the characteristics of particle motion. M. Hatami and G. Domairry [6] studied the unsteady settling behavior of a soluble spherical particle falling in a Newtonian fluid media by DTM–Padé. They discussed about the influence of the solubility parameter on velocity profile. M. Jalaal and D.D. Ganji [7] studied the unsteady motion of a spherical particle rolling down an inclined plane submerged in a Newtonian environment using a drag of the form given by R.P. Chhabra and J.M. Ferreira [8], for a wide range of Reynolds numbers by the homotopy perturbation method (HPM). They observed that the settling velocity, acceleration duration and displacement are proportional to the inclination angle, while for a constant inclination angle, the settling veloc-ity and acceleration duration are decreased by increasing the fluid viscosity. M. Jalaal et al. [9] applied the HPM to obtain exact analytical solutions for the motion of a spherical particle in a plane Couette flow. M. Jalaal et al. [10] applied the variational iteration method (VIM) on the acceleration motion of a non-spherical particle in an incompressible Newtonian environment for a wide range of Reynolds numbers using a novel drag coefficient as defined by S.F. Chien [11]. In [12,13] the unsteady motion of a spherical particle falling in a Newtonian fluid was analyzed using the HPM. M. Torabi et al. [14] applied HPM–Padé approximant method to obtain exact analytical

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solutions for acceleration motion of a vertically falling spherical particle. S.M. Hamidi et al. [15] applied HPM–Padé approximant method to obtain exact analytical solutions for the motion of a spherical solid particle in the plane Couette fluid flow.

The concept of Differential Transformation Method (DTM) was first introduced by J.K. Zhou [16] and it was used to solve both linear and nonlinear initial value problems. This method can be applied directly to linear and nonlinear differential equation without requiring linearization, discretization, or perturbation and this is the main benefit of this method. T. Abbasov, A.R. Bahadir [17] employed DTM to obtain approximate solutions of the linear and non-linear equations related to engineering problems and they showed that the numerical results are in good agreement with the analytical solutions. M.M. Rashidi et al. [18] solved the problem of mixed convection about an inclined flat plate embedded in a porous medium by DTM; they applied the Padé approximant to increase the convergence of the solution. S. Ghafoori et al. [19] used the DTM for solving the nonlinear oscillation equation. I.H. Abdel-Halim Hassan [20] has applied the DTM for different systems of differential equations and he has discussed the convergency of this method in several examples of linear and non-linear systems of differential equations.

The goal of this study is obtaining an analytical solution for governing equations of a spherical solid particle's motion and particle motion analysis in the plane Couette fluid flow. Also the Differential Transformation Method (DTM)-Padé is applied to solve linear problems analytically. To validate analytical results, the obtained DTM–Padé results are compared with numerical data.

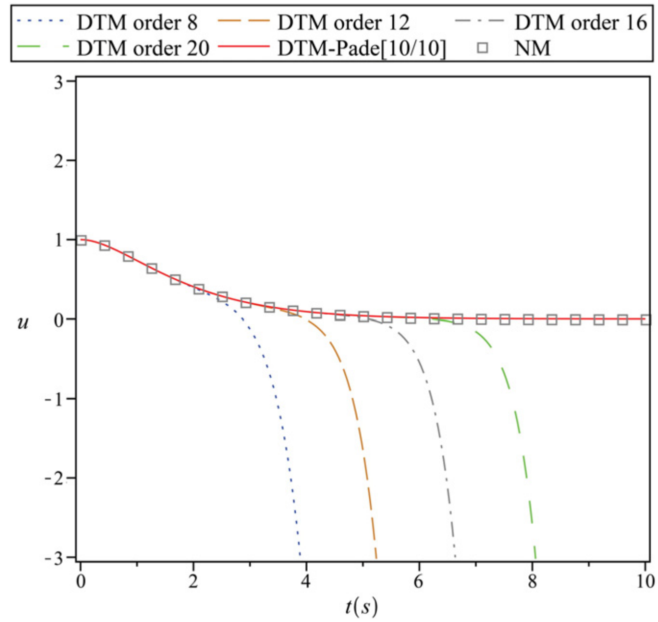

Fig. 1. Comparison of horizontal velocity (u) obtained by DTM and DTM–Padé with numerical solution when $A = B = C = \alpha = u_0 = v_0 = 1$.

2. Problem description

T.J. Vander Werff [21] proposed a constructive mathematical formulation for the motion of a spherical particle in a plane Couette flow. T.J. Vander Werff [21] assumed a two-dimensional velocity profile incompressible Newtonian flow. Considering the rotation of the particle, it was assumed that the particle will rotate with a constant angular velocity Ω given by one-half the curl of the fluid motion. Generally, the particle's motion is determined by the combined effects of inertia, drag and lift. Gravity and buoyancy effects will be assumed negligible [21]. Subsequently, the governing equations are obtained as:

$$\begin{cases} \frac{4}{3}\pi r^3 \rho_s \ddot{x} = \frac{1}{2}\pi r^3 \rho_f \alpha \, \dot{y} - 6\pi\mu r(\dot{x} - \alpha y) \\ \frac{4}{3}\pi r^3 \rho_s \ddot{y} = \left(\frac{1}{2}\pi r^3 \rho_f \alpha + 6.46 r^2 \rho_f \alpha^{1/2} \nu^{1/2}\right)(\alpha y - \dot{x}) - 6\pi\mu r \dot{y} \end{cases} \tag{1}$$

where r, ρ_s and ρ_f represent the particle radius, particle density and fluid density, respectively. Also, μ and ν indicate the dynamic fluid viscosity and kinematic fluid viscosity, respectively and α is a positive proportionality constant with dimensions of inverse time. Moreover, the dots represent differentiation with respect to time. The relative velocities of the particle and fluid are considered small enough for the Stokes law of drag [21]. For this system to possess a nontrivial solution, nonzero initial conditions must be specified. The following might represent either injection of the particle into the fluid or statistical fluctuations: Alternatively, nonzero values of x or y could have been chosen where no generality is lost by specifying the particular (and physically more meaningful) conditions above. Eq. (1) can be rewritten in the following forms:

$$\begin{cases} \ddot{x} - A\,\dot{y} + B(\dot{x} - \alpha y) = 0 \\ \ddot{y} + B\,\dot{y} + (A + C)(\dot{x} - \alpha y) = 0 \end{cases} \tag{2}$$

Table 1
The fundamental operations of the differential transform method.

Original function	Transformed function
$w(t) = \alpha u(t) \pm \beta v(t)$	$W(k) = \alpha U(k) \pm \beta V(k)$
$w(t) = \frac{d^m u(t)}{dt^m}$	$W(k) = \frac{(k+m)!}{k!} U(k+m)$
$w(t) = u(t)v(t)$	$W(k) = \sum_{l=0}^{k} U(l)V(k-l)$
$w(t) = t^m$	$W(k) = \delta(k-m) = \begin{cases} 1, & \text{if } k = m \\ 0, & \text{if } k \neq m \end{cases}$
$w(t) = \exp(t)$	$W(k) = \frac{1}{k!}$

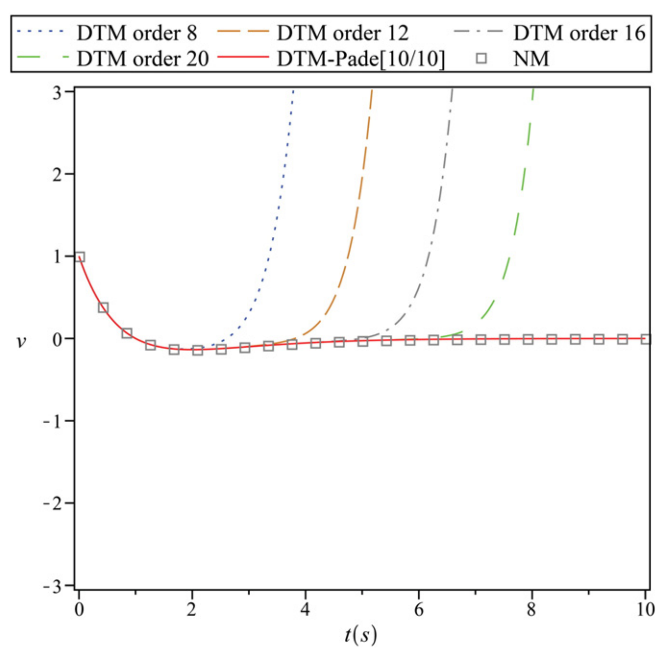

Fig. 2. Comparison of vertical velocity (v) obtained by DTM and DTM–Padé with numerical solution when $A = B = C = \alpha = u_0 = v_0 = 1$.