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Unsteady flow and heat transfer of pseudo-plastic nanoliquid in a finite thin film on a stretching surface with variable thermal conductivity and viscous dissipation

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ABSTRACT

This paper studies flow and heat transfer of pseudo-plastic nanoliquid in a finite thin film over an unsteady stretching surface with variable thermal conductivity and viscous dissipation effects. Four different types of nanoparticles, Cu, Al_2O_3, CuO and TiO_2 are considered with sodium carboxymethyl cellulose (CMC)-water used as a base fluid. Unlike most classical works, a modified Fourier's law of heat conduction for power-law fluids is adopted by assuming that the thermal conductivity is power-law-dependent on the velocity gradient. Similarity transformations are applied to reduce the governing partial differential equations into a system of nonlinear ordinary differential equations, which are solved numerically by a shooting method coupled with Runge–Kutta method and BVP4C. The effects of solid volume fraction, types of nanoparticles, power-law index, unsteadiness parameter, modified Prandtl number and Eckert number on film thickness, velocity and temperature fields are graphically illustrated and discussed.

1. Introduction

The flow and heat transfer of a thin film over a stretching sheet have attracted much attention due to its applications arising from many fields of science and technology. Such applications include wire and fiber coating, metal and polymer extrusion, foodstuff processing, continuous casting, drawing of plastic sheets, exchangers, transpiration cooling, reactor fluidization, chemical processing equipment, etc. All coating processes demand a smooth glossy surface to meet requirements for appearance, low friction, transparency and strength. The analysis of flow and heat transfer within a thin film on a continuously stretching surface is important. In view of these applications, Wang [1] studied the flow of a Newtonian fluid within a finite thin film over an unsteady stretching sheet. It showed that the exact similarity solutions of the unsteady thin film Navier–Stokes equations may be obtained by restricting the motion to a specified family of time dependence. Later, Andersson et al. [2] studied the flow of an incompressible fluid in a thin liquid film obeying a power law model. Afterwards, Andersson and coworkers [3,4] explored the heat transfer characteristics of the hydrodynamical problem solved by Wang [1]. Chen [5–7] examined the flow and heat transfer of power law fluid in a thin liquid film on an unsteady surface and took the effects of viscous dissipation or Marangoni convection into account. Wang and Pop [8] considered the flow of a power law fluid film on an unsteady stretching sheet. Analytical solutions are obtained using the homotopy analysis method (HAM) and a critical value for unsteadiness parameter is derived. Abel and coworkers [9,10] studied MHD flow and heat transfer of a laminar liquid film over an unsteady stretching surface with external magnetic field and viscous dissipation effects. Aziz and coworkers [11–13] extended the problem of flow and heat transfer of finite thin film over an unsteady stretching sheet by considering a general surface temperature. A list of key references in the literature concerning this field can be found in Refs. [14–20].

All of the abovementioned investigations are restricted to pure fluids (Newtonian or non-Newtonian). As a novelty, nanofluid, proposed by Choi [21], is physically described as a heat transfer basic fluid containing a suspension of submicronic solid particles (nanoparticles). It is expected that a mixture of the base fluid and nanoparticles will develop advanced heat transfer fluids with higher conductivities. Wang and Mujumdar [22] and Haddad et al. [23] offered an overview of the literature on the recent developments in nanofluids. The problems of flow and heat transfer within a finite thin film over an unsteady stretching sheet are extended by Bachok et al. [24], Xu et al. [25] and Narayana and Sibanda [26] to nanoliquid films. It should be noted that

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the base fluids of nanoliquid are restricted to Newtonian (pure) fluids in Refs. [24–26].

Pseudo-plastic non-Newtonian fluids are important due to its many industrial applications. Recently, considerable attention has been devoted to the problem of predicting the behavior of non-Newtonian fluids. Pop and coworkers [27,28] proposed a model that the thermal conductivity of power law fluids was power-law-dependent on the velocity gradient. Chamkha [29] analyzed a steady laminar boundary layer flow and heat transfer in a quiescent non-Newtonian fluid driven by a stretched porous surface. The flow and heat transfer of power law fluids are further studied by considering various thermal conductivity models such as Pop's or Zheng's by Lin et al. [30–33].

In this paper we investigate the flow and heat transfer of pseudo-plastic nanoliquid in a finite thin film over an unsteady stretching surface with variable thermal conductivity and viscous dissipation effects. We assume that the temperature field is similar to the velocity and the thermal conductivity of the fluid is power-law-dependent on the velocity gradient by modified Fourier's law. Four types of nanoparticles, i.e., copper (Cu), aluminum oxide (Al_2O_3), copper oxide (CuO), and titanium oxide (TiO_2) are considered. The CMC-water (carboxyl methyl cellulose) is used as the base fluid of nanoliquid [33]. The partial differential governing equations are reduced and solutions are obtained numerically by BVP4C and the shooting method coupled with the Runge–Kutta method. The effects of solid volume fraction, power-law index, unsteadiness parameter, modified Prandtl number and Eckert number are illustrated graphically via the velocity and temperature profiles.

The novel contributions of this paper are the following:

(i) Pseudo-plastic nanoliquid constitutive law modeling by incorporating import physical effect;
(ii) The consideration of finite thickness of the thin film with surface stretching;
(iii) The distinctive resulting properties of various nanoparticles.

2. Formulation for stretching surface problem

Table 1
Viscous properties of base fluid (CMC-water) [32,33].

Physical properties	CMC-water (0.0%)	CMC-water (0.1%)	CMC-water (0.2%)	CMC-water (0.3%)	CMC-water (0.4%)
n	1.00	0.91	0.85	0.81	0.76
K (Ns^n/m^2)	0.000855	0.006319	0.017540	0.0313603	0.0785254

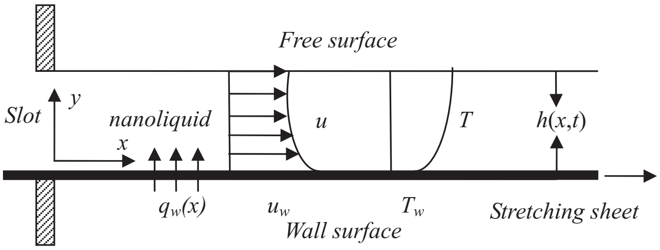

Fig. 1. Schematic of the physical system.

Consider the flow and heat transfer of pseudo-plastic nanoliquid in a finite film over a horizontal sheet issuing from a narrow slot. The fluid motion is caused by the stretching of the elastic sheet. A schematic of the physical model and coordinate system is shown in Fig. 1. The CMC-water-based nanoliquid contains different types of nanoparticles. Experimental studies show that the carboxymethyl cellulose (CMC) water solution with concentration of 0.0%–0.4% can be approximated as pseudo-plastic fluids with power-law index $0 < n < 1$ (see the comment after Eqs. (2) and (3)). In this study, the CMC-water with concentration (0.1%–0.4%) is used as a base fluid of nanoliquid. The viscous properties of the CMC-water are given in Table 1 [33]. Thermophysical properties of the nanoliquid are given in Table 2 [33–37]. We assume that the nanoliquid is incompressible, the flow is laminar, the base fluid and the nanoparticles are in thermal equilibrium and that no slippage occurs between them. Furthermore, it is assumed that the stretching of elastic sheet has viscous dissipation, the interface of the nanoliquid film is smooth, viscous shear stress and heat flux vanish at the adiabatic free surface. Under these assumptions, the governing conservation equations of mass, momentum and energy can be expressed as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial}{\partial y}\left(\frac{\mu_{nf}}{\rho_{nf}}\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right), \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha_{nf}\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial T}{\partial y}\right)+\frac{\mu_{nf}}{\left(\rho C_p\right)_{nf}}\left|\frac{\partial u}{\partial y}\right|^{n-1}\left(\frac{\partial u}{\partial y}\right)^2. \tag{3}$$

Note that the coefficient term $\frac{\mu_{nf}}{\rho_{nf}}\left|\frac{\partial u}{\partial y}\right|^{n-1}$ in Eq. (2), $\alpha_{nf}\left|\frac{\partial u}{\partial y}\right|^{n-1}$ and $\frac{\mu_{nf}}{(\rho C_p)_{nf}}\left|\frac{\partial u}{\partial y}\right|^{n-1}$ in Eq. (3) model the pseudo-plastic property of the fluid. The boundary conditions are

$$y=0: u=u_w, v=0, T=T_w, \tag{4}$$

$$y\to h(x,t): \frac{\partial u}{\partial y}=\frac{\partial T}{\partial y}=0, v=u\frac{\partial h}{\partial x}+\frac{\partial h}{\partial t}, \tag{5}$$

where u and v are the velocity components along the x and y directions, respectively, t is time, $\tau_{xy}=\mu_{nf}|\partial u/\partial y|^{n-1}\partial u/\partial y$ is shear stress, $\mu=\mu_{nf}|\partial u/\partial y|^{n-1}$ is effective viscosity of nanoliquid, μ_{nf} is modified consistency viscosity coefficient, ρ_{nf} is effective density of nanoliquid and n is power law index. The case $n=1$ corresponds to a Newtonian fluid, $0<n<1$ describes a pseudo-plastic fluid while $n>1$ is for a dilatant fluid. T is temperature, C_p is specific heat at constant pressure, $(\rho C_p)_{nf}$ is effective heat capacity of the nanoliquid, and $h(x, t)$ is thickness of the nanoliquid film. It is noted that the condition $v=u\frac{\partial h}{\partial x}+\frac{\partial h}{\partial t}$ imposes a kinematic constraint of the fluid motion [1–7]. The effects of power law viscosity on temperature fields are taken into account by assuming that the temperature field is similar to the velocity field and the thermal conductivity is power-law-dependent on velocity as $k=\alpha_{nf}(\rho C_p)_{nf}|\partial u/\partial y|^{n-1}$ (k is the effective thermal conductivity of the nanoliquid, α_{nf} is the modified thermal diffusivity of the nanoliquid) and $\alpha_{nf}|\partial u/\partial y|^{n-1}$ is the effective thermal diffusivity of nanoliquid. The term $\mu_{nf}/(\rho C_p)_{nf}|\partial u/\partial y|^{n-1}(\partial u/\partial y)^2$ accounts for the viscous dissipation effect. The flow is caused by stretching the wall surface (the elastic surface) at $y=0$ such that the continuous sheet moves in the x-direction with the velocity u_w as

$$u_w=bx/(1-at), \tag{6}$$

Table 2
Thermophysical properties of base fluid and nanoparticles [32–37].

Physical properties	CMC-water (0.0%–0.4%)	Cu	Al_2O_3	CuO	TiO_2
C_p (J/kg K)	4179	385	765	535.6	686.2
ρ (kg m^{-3})	997.1	8933	3970	6500	4250
k (W/m K)	0.613	400	40	20	8.9538