Powder Technology 277 (2015) 222–230

Contents lists available at ScienceDirect

Powder Technology

journal homepage: www.elsevier.com/locate/powtec

A new shape dependent drag correlation formula for non-spherical rough particles. Experiments and results

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ARTICLE INFO

Article history:
Received 16 December 2014
Received in revised form 2 February 2015
Accepted 28 February 2015
Available online 16 March 2015

Keywords:
Drag coefficient
Shape factor
Terminal velocity
Non-spherical particles
Multiphase flow
Settling

ABSTRACT

The drag of non-spherical rough particles has been investigated in a wide range of Reynolds numbers (0.03–10,000). The study is based on experimental measurements of the terminal velocities of irregular particles falling in fluids of different densities and viscosities. The particle shape is described by a shape factor that takes into account both sphericity and circularity, which are measured via image particle analysis techniques. This shape factor is particularly suitable for non-spherical and highly irregular particles. The drag coefficient has been correlated to the particle Reynolds number and the shape factor and a new correlation law has been found; the correlation has the functional form of a power law. Due to the mutual dependency of the particle terminal velocity on the drag coefficient, which in turn depends on the particle shape and Reynolds number, an iterative procedure needs to be designed for calculating the terminal velocity of particles of a specific size and shape. Such a procedure is adopted herein and a spreadsheet and a Fortran 90 code allowing the iterative calculation are provided in the Supplementary Material. The fitting of experimental measurements with our model calculations show that our new law predicts the drag coefficients and the terminal velocity of irregularly shaped particles, as volcanic ash, more accurately than other shape-dependent drag laws.

1. Introduction

The knowledge of how irregularly shaped particles settle in quiescent or moving fluids is fundamental in a wide range of research fields and applications, from industrial (e.g. chemical engineering) to natural processes (sedimentation of solid particles in rivers and during explosive eruptions) [1–6]. The equilibrium velocity w_t at which particles settle in a static Newtonian liquid can be calculated by balancing the surface (drag) and body forces acting on the particle [7], thus obtaining the Netwon's impact law:

$$w_t = \sqrt{\frac{4gd_p\left(\rho_p - \rho\right)}{3C_d\rho}} \tag{1}$$

where g is the gravitational acceleration, d_p is the particle size, ρ_p is the particle density, ρ is the fluid density and C_d is the drag coefficient (refer to Table 1 for Symbol notation), which, for a spherical particle, is solely a function of the particle Reynolds number:

$$Re = \frac{\rho w_t d_p}{\mu} \tag{2}$$

where μ is the fluid viscosity. From numerous experiments on spherical particles it was possible to define a C_d vs. Re experimental relationship, which was however not easily interpolated to obtain a single drag law over the entire Re range (from 0 to 10^7). For this reason several correlations have been proposed in the literature, each having a different accuracy and range of applicability (e.g. Schiller and Naumann [8]; Dallavalle [9]; Clift et al. [10]; White [11]). For non-spherical particles these correlations are no longer valid and the drag coefficient is a function of both particle Reynolds number and shape.

In the multiphase fluid dynamics literature there is not a unique way to define the shape of a particle. Some authors use the ratio between the volume-equivalent-sphere and the surface-equivalent-sphere diameters (d_n/d_A), where the surface is that of the projected area [2,12–14]. Other authors, in order to focus on the effects of particle elongation, define particles shape in terms of the aspect ratio (particle length along the symmetry axis over the largest diameter of the cross section) [15,16]. Another important shape factor is the sphericity, which is the ratio between the surface area of the equivalent sphere A_{sph} and that of the actual particle A_p:

$$\Phi = \frac{A_{sph}}{A_p}. \tag{3}$$

Sphericity is probably the most widely used among the shape factors, as it is generally considered to be an accurate shape descriptor

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http://dx.doi.org/10.1016/j.powtec.2015.02.062
0032-5910/

for isometric non-spherical particle [3,17–21]. However it is very difficult to be evaluated for highly irregular particles, since it is not easy to measure A_p. Therefore other shape factors have been devised, e.g. the circularity c [2]:

$$c = \frac{\pi d_A}{P} \tag{4}$$

where P is the projected perimeter of the particle in its direction of motion. Another possibility is to use combinations of the three principal axes of an irregular particle (d_l, d_m, d_s: long, medium and short axes, respectively), e.g. the Corey shape factor [22,23]. Finally, for highly irregular particles, Dellino et al. [3] found that the ratio between sphericity Φ and a newly defined circularity X was an effective shape descriptor:

$$\Psi = \frac{\Phi}{X}. \tag{5}$$

The circularity X is more suitable in the case of experiments in which the secondary motions of the falling particles are not taken into account in the analysis. Indeed parameters like c (Eq. (4)) can be measured in experiments in which the particle shape is regular, although not spherical, thus allowing to have a complete and constant control of its position relative to the surrounding fluid during the fall experiment and, therefore, to know the value of the projected area and perimeter. Details on Ψ will be discussed in the following section.

Basing on these diverse shape factors and on experimental data from free-fall experiments (e.g. Tran-Cong et al. [2]; Dellino et al. [3]), wind tunnel testing (e.g. Bagheri et al. [24]) or by compiling data from the literature, previous authors (e.g. Chhabra et al. [1]; Hölzer and Sommerfeld [21]) have found different correlations between C_d, Re and particle shape. Generally these relationships are valid among specific Re ranges, and are obtained from experiments on non-spherical particles but with well-defined shapes (e.g. cylinder, disks, octahedrons, etc.), which allow a good control on the effective area exposed by the particle to the fluid during settling. However, this case is not representative of irregular natural cases, such as those transported by wind or water or pumice particles produced during explosive eruptions, in which the grains are far to be described by well-defined shapes. In this sense the law of Dellino et al. [3] represented a useful improvement, as it was derived from free-falling experiments of highly irregular particles of volcanic origin of different sizes, densities and shapes. The relationship found by Dellino et al. [3] is valid for $Re > 50$, although it was later verified that its validity could be extended down to 10 [25]. In this paper new experiments on the same particles falling at different Re numbers were designed in order to cover lower Re values down to less than 1 and a new drag law covering the enlarged Re range was obtained. This represents, in our opinion, an important step forward for the characterization of fine particulate solids transportation by fluid currents.

2. Particle shape factors and settling experiments

In order to obtain a model to predict the terminal velocity of irregularly shaped particles, the particle parameters have been measured and experiments performed on them following the procedure described in Dellino et al. [3] and Mele et al. [26]. The complete experimental dataset is reported in the Microsoft Office Excel file table.xlsx (in the first sheet "Experiments") available as Supplementary Material.

2.1. Particle parameters

Experiments were performed using the same set of particles employed in Dellino et al. [3]. The particles were sampled from the volcanic material erupted during explosive eruptions at Vesuvius and Campi Flegrei volcanoes, Italy. They are characterized by a wide range of size, density and shape.

Table 1
Symbol notation.

Symbol	Description	Units
Latin		
a	Exponent applied to Re	–
A_p	Surface area of the actual particle	cm^2
A_{sph}	Surface area of the equivalent sphere	cm^2
c	Particle circularity (as defined in [2])	–
C_d	Fluid-particle drag coefficient	–
$C_{d,norm}$	Normalized fluid-particle drag coefficient	–
$C_{d,sphere}$	Drag coefficient of the spherical particle	–
d_A	Surface-equivalent-sphere diameter	cm
d_l	Long axis of the particle	cm
d_m	Medium axis of the particle	cm
d_n	Volume-equivalent-sphere diameter	cm
d_p	Particle dimension	cm
d_s	Small axis of the particle	cm
$err\%$	Relative error of $w_{t,calc}$ with respect to $w_{t,meas}$	–
exp	Exponent applied to the particle shape factor	–
g	Gravitational acceleration	cm s^{-2}
K_1	First Ganser's shape factor	–
K_2	Second Ganser's shape factor	
m	Particle mass	g
P	Projected perimeter of the particle in its direction of motion	cm
P_{mp}	Maximum projection particle perimeter	cm
P_p	Perimeter of the circle equivalent to the maximum projection area	cm
Re	Particle Reynolds number	–
Re^*	Initial guess for particle Reynolds number in the iterative procedure	–
X	Particle circularity	–
w_t	Terminal velocity	cm s^{-1}
$w_{t,calc}$	Terminal velocity calculated by drag laws	cm s^{-1}
$w_{t,meas}$	Terminal velocity measured during the experiments	cm s^{-1}
$w_{t,Stokes}$	Terminal velocity calculated with the Stoke's law	cm s^{-1}
Greek		
β	Corey shape factor	–
μ	Fluid viscosity	P
ρ	Fluid density	g cm^{-3}
ρ_p	Particle density	g cm^{-3}
Φ	Particle sphericity	–
Ψ	Particle shape factor	–

The size of a particle, d_p, was taken as equal to the diameter of the equivalent sphere, d_n:

$$d_p = d_n = \sqrt[3]{\frac{6m}{\pi \rho_p}} \tag{6}$$

where m is the particle mass. Particle density ρ_p was measured, depending on the particle size, by two standard Gay–Lussac picnometers of 25 and 5 ml capacities.

In order to calculate particle sphericity Φ, which is needed for the evaluation of Ψ, A_{sph} and A_p had to be measured (Eq. (3)). The surface area of the equivalent sphere A_{sph} is:

$$A_{sph} = 4\pi \left(\frac{d_p}{2}\right)^2. \tag{7}$$

As an approximation of A_p, we took the area of a scalene ellipsoid [3]:

$$A_p = 4\pi \left(\frac{(d_l/2)^2(d_m/2)^2 + (d_l/2)^2(d_s/2)^2 + (d_m/2)^2(d_s/2)^2}{3}\right)^{\frac{1}{1.6075}} \tag{8}$$

where d_l, d_m, d_s were defined in the Introduction section.