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Short communication

Fluid–solid interfacial drag force on monodisperse assemblies of spherical particles

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ABSTRACT

The mathematical up-scaling of flow past an assembly of spherical particles was analyzed. From an averaged model, a closure problem for the interfacial force obtained *via* the volume averaging method with closure was solved in cylindrical unit cells with concentric particles. In concordance with reported works in the framework of solid–fluid flows, the interfacial force was expressed in terms of a drag coefficient. Comparisons with available data from direct numerical simulations were in good agreement, and a correlation that fits the calculated data for drag coefficient was proposed. Finally, some remarks on advantages and limitations of the applied approach are made.

1. Introduction

The ability to predict the behavior of transport processes in solid–fluid flows is a complex problem for physics and applied mathematics. The averaged models [1,2] overcome the problem that the spatial discontinuities of phases and the displacement of interfaces imply; they are continuous over the entire domain, but at the expense of the occurrence of closure terms. These latter involve filtered information due to spatial smoothing applied by averaging procedures when the up-scaled equations are deduced.

Regarding the closure terms in averaged models for particulate two-phase flows, many researchers use a practical approach based on the direct usage and up-scaling of mathematical solutions for flow around a disperse-phase representative body. It is interpreted that all the dispersed bodies in the system undergo the same phenomena governed by invariant constitutive equations [3]. This approach is supported by the principle of material objectivity [4,5]. The precise definition of closure parameters for this approach often implies an associated unit cell problem, complemented by empirics and/or numerical findings [3,4,6]. One alternative to this is the volume averaging method with closure (VAMC), proposed by Whitaker [1]. This method goes beyond the deduction of average equations, since it proposes a mathematical formalism to deduce a set of boundary-value problems associated to the closure terms.

The VAMC begins with the application of averaging operators and theorems to local conservation equations over one averaging region of multiphase systems. Then, the remaining local variables in the averaged equations are substituted by their spatial decompositions (the average variables plus their deviation around these average values). The closure terms are associated to averages including deviation variables that cannot be neglected even if analysis of magnitude orders are carried out in terms of reasonable scale constrains. Following, the averaged equations are subtracted from the local ones, given as result a differential problem in terms of deviation and average variables. If the length scale for deviation variables characterizes the size of one representative and periodic region, the terms involving average variables can be assumed as constants at that periodic region, this as long as their characteristic lengths of variation be much larger than the corresponding ones involving deviation variables. Therefore, the subtracted system results in a problem which must be solved only for the deviation variables [1,7,8].

The study of fluidization of mono-dispersed particles or flow past assemblies of particles are useful for the fundamental understanding of transport phenomena which are implicated in solid–fluid particulate flows [9]. This approach has been applied in the analysis based on DNS [10–14], a conciliator DNS-average model analysis [15], and also it has been employed for developing correlations in early works [9]. The validity of this approach is tied to high Stokes numbers found in several engineering applications [10,15]. In this work, the mathematical up-scaling of flow past an assembly of monodisperse spherical particles by means of the VAMC was analyzed, with a focus on the interfacial

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force term. The matching of VAMC with a conventionally employed approach for the closure definition in particulate flows was a matter of interest.

2. Averaged model and closure problem

Let us consider the flow in the bulk of a Newtonian and incompressible fluid (f) through an assembly of dispersed particles; a representation of this system is given in Fig. 1. L is a characteristic macroscopic length and l_f is a characteristic length where significant local changes occur. The boundary-value problem describing the relative fluid flow with respect to the assembly of solids at the local scale is the following:

$$\rho_f\left(\frac{\partial \mathbf{v}_f}{\partial t'}+\mathbf{v}_f\cdot\nabla'\mathbf{v}_f\right)=-\nabla' p_f+\mu_f\nabla^2\mathbf{v}_f+\rho_f\mathbf{g}, \tag{1}$$

$$\nabla'\cdot\mathbf{v}_f=0, \tag{2}$$

$$\mathbf{v}_f=0 \quad \text{at } A_{sf}, \tag{3}$$

$$\mathbf{v}_f=\boldsymbol{G}(\mathbf{r}',t') \quad \text{at } \mathrm{A}_{fe}, \tag{4}$$

$$\mathbf{v}_f=\boldsymbol{J}(\mathbf{r}') \quad \text{at } t'=0. \tag{5}$$

Here, ρ and μ are the density and dynamic viscosity, respectively. The interfacial area between phases is identified by A_{sf}, while A_{fe} represents the area of entrances to the global system for the fluid phase.

If gradients of volumetric fluid fraction (ε_f) are neglected, the following model is obtained after averaging:

$$\nabla\cdot\left\langle\mathbf{v}_f\right\rangle^f=0, \tag{6}$$

$$\rho_f\left(\frac{\partial\left\langle\mathbf{v}_f\right\rangle^f}{\partial t}+\left\langle\mathbf{v}_f\right\rangle^f\cdot\nabla\left\langle\mathbf{v}_f\right\rangle^f\right)=-\nabla\left\langle p_f\right\rangle^f+\mu_f\nabla^2\left\langle\mathbf{v}_f\right\rangle^f+\rho_f\mathbf{g}-\frac{\rho_f}{\varepsilon_f}\nabla\cdot\left(\varepsilon_f\left\langle\tilde{\mathbf{v}}_f\tilde{\mathbf{v}}_f\right\rangle^f\right)+\frac{\mathbf{M}_{fs}}{\varepsilon_f}, \tag{7}$$

subject to the following scale constraints:

$$l_f<<d<<L, \tag{8}$$

and being

$$\mathbf{M}_{fs}=\frac{\varepsilon_f}{TV_f}\int_{t-T/2}^{t+T/2}\int_{A_{sf}(\mathbf{x},t)}\mathbf{n}_{fs}\cdot\left(-\tilde{p}_f\mathbf{I}+\mu_f\nabla\tilde{\mathbf{v}}_f\right)dAdt',$$

an interfacial force closure [2,5]. d is a characteristic length of the averaging region, the operator $\langle\bullet_f\rangle^f$ is the intrinsic volume average, $\tilde{\bullet}$ denotes a deviation variable; V_f and T are the volume occupied by f-phase and the time in a spatial and temporal averaging region, respectively.

The following closure problem was derived by Whitaker [7] in the framework of study of flow through porous media:

$$\nabla\cdot\mathbf{A}_f=0 \quad \text{en } V_f, \tag{9}$$

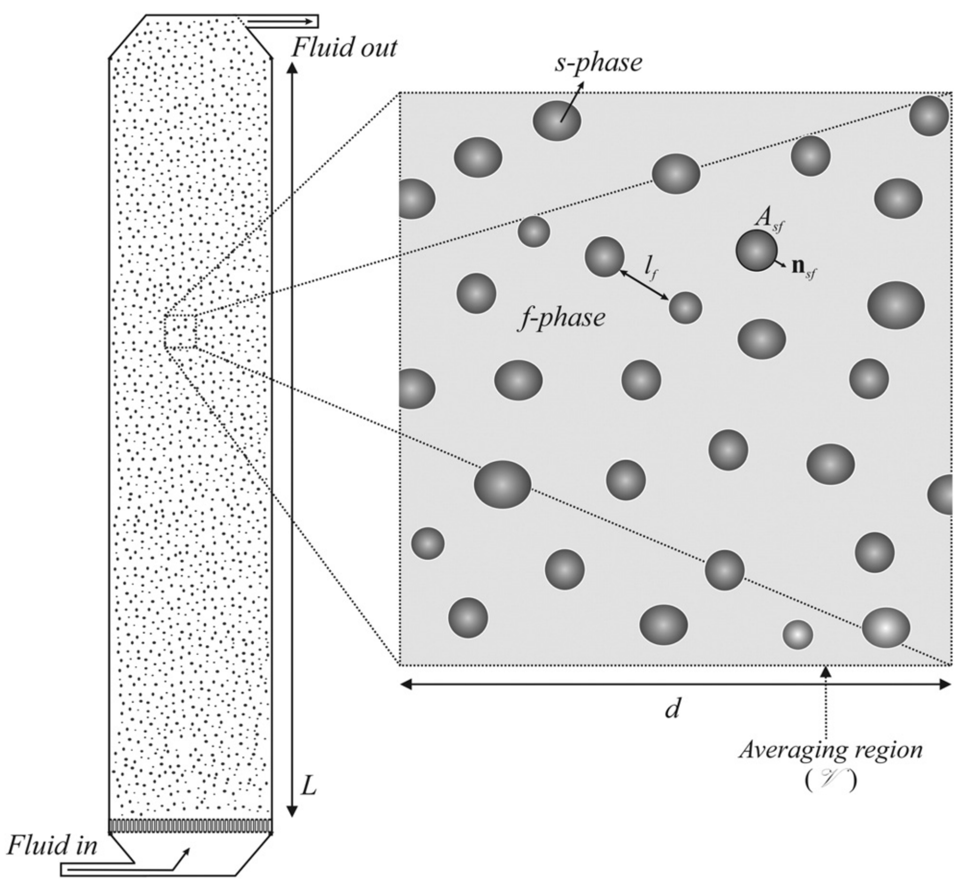

Fig. 1. A geometrical representation of a uniform particulate solid–fluid two-phase flow.