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Application of pseudo-fluid approximation to evaluation of flow velocity through gravel beds

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ABSTRACT

Flows seeping through a gravel bed are usually non-Darcian and closely related to non-linear drag. Such flows may be significantly affected by particle shape and bed configuration. In this study, a pseudo-fluid model is developed to calculate average flow velocity through gravel beds. The proposed approach is able to take into account particle shape effect using the drag coefficient associated with an isolated sediment grain and also bed configuration effect in terms of apparent viscosity. The model was then calibrated with ten series of laboratory data, which were collected using vertical columns packed with spherical and natural gravels. Finally, the model was successfully applied to estimate total flow discharges for laboratory-scale open channel flows over a gravel bed.

1. Introduction

A flow system of particle–fluid mixture may be treated as a single phase characterized with apparent density and viscosity, which would yield a pseudo-fluid model. Such models have been successfully applied in the description of characteristics of various particle–fluid mixtures, for example, in studying fluidization [1] and the transport of high-concentrated sediment [2]. Cheng [3] also shows that the hindered settling velocity of sediment particles could be well estimated based on the pseudo-fluid concept.

Although the pseudo-fluid approximation usually applies for particle–fluid mixtures of which both phases are mobile, it could also be extended to flow passing through fixed solid phase. Such an attempt was recently reported by Cheng [4], who developed a pseudo-fluid approach to estimate the drag coefficient for cylinder-simulated vegetation stems presented in open channel flows. To derive the approach, an analogy was made between the channel flow through vegetation stems and the settling of a cylinder array, which provides an effective connection between the parameters used in the pseudo-fluid model and those measurable for open channel flows subject to the simulated vegetation. The result obtained by Cheng [4] shows that the relationship between drag coefficient and Reynolds number, which applies for an isolated cylinder, could be generalized for evaluation of the drag coefficient for one cylinder in an array. The present study aims to develop a similar method to calculate flow velocity through a sediment bed comprised of immobile gravels.

Flows passing through a sediment bed composed of gravels are usually non-Darcian, as observed in flows through other coarse materials like rockfills and waste dumps. Non-Darcian flows are closely related to nonlinear drag. Some theoretical attempts have been devoted to associate the nonlinear drag with inertial and/or turbulent effects of viscous flow [5–7]. However, the current understanding of relevant flow phenomena is limited and thus it is still challenging to theoretically describe non-Darcian flows [8]. On the other hand, it is noted that Darcy law could be extended to flows with significant inertial effects through Ergun equation [9], which relates the hydraulic gradient to the flow velocity in the quadratic form,

$$S = a_E \frac{\nu(1-\varepsilon)^2}{gD^2\varepsilon^3} V + b_E \frac{1-\varepsilon}{gD\varepsilon^3} V^2 \quad (1)$$

where $a_E = 150$, $b_E = 1.75$, S is the hydraulic gradient, ν is the kinematic viscosity of fluid, ε is the porosity, ρ is the fluid density, g is the gravitational acceleration, D is the grain diameter and V is the superficial flow velocity calculated as the ratio of the flow rate to the bulk cross-section area. The Ergun equation suggests that the energy loss can be computed simply by summing up the two components, one being caused by the viscous effect and the other due to the inertial effect [10]. Moreover, recent experimental and numerical studies show that the deviation from Darcy's law could be closely associated with formation of a viscous boundary layer, the interstitial drag force, separation

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of flow, or formation of eddies [11,12]. These explanations serve as good qualitative description of the inertia-affected flow field, but each of them is in itself a challenging task in providing quantitative connections with non-linear flow characteristics. Therefore, further efforts are needed to explore physics of non-Darcy flow in depth.

By implementing the pseudo-fluid concept, this study aims to provide an alternative consideration of the complicated non-linear drag without looking into complicated flow phenomena inside pores. The paper is outlined as follows. First, the pseudo-fluid approximation is applied to quantify bulk properties of the flow through a packed bed. Then, the resulted pseudo-fluid model is calibrated using experimental data. Comparisons are also made between the predictions by the present model and Ergun equation. Finally, it is shown that the model can be used to estimate the total flow discharge for laboratory-scale open channel flows over a gravel bed.

2. Pseudo-fluid approximation

To apply the pseudo-fluid concept, we start with the terminal velocity of a single particle settling in a stationary fluid. For this case, the effective weight of the particle is equal to the drag induced by its downward motion relative to the fluid. The drag is expressed as

$$F_D = C_D \frac{\pi D^2}{4}\frac{\rho w^2}{2} \tag{2}$$

where C_D is the drag coefficient, and w is the settling velocity. The effective weight of the particle is

$$W = (\rho_s - \rho) g \frac{\pi D^3}{6} \tag{3}$$

where ρ_s is the particle density. Under the terminal condition, $F_D = W$, and thus with Eqs. (2) and (3),

$$C_D = \frac{4}{3}\frac{\Delta g D}{w^2} \tag{4}$$

where $\Delta = (\rho_s - \rho) / \rho$ is the relative density difference.

It is noted that C_D generally varies with Reynolds number Re defined as wD / ν. When the settling occurs in the Stokes regime, e.g. for Re < 1, C_D is linearly proportional to 1/Re. In the inertial regime, e.g. for Re > 1000, the viscous effect is insignificant and C_D can be approximated as a constant. In between the two regimes, the dependence of C_D on Re is complex. In the literature, many empirical formulas have been proposed to describe the relationship of C_D and Re in a wide range of Re [13–16]. However, for simplicity, the variation of C_D with Re can be approximated using the following three-parameter formula,

$$(C_D)^m = \left(\frac{M}{Re}\right)^m + (N)^m \tag{5}$$

where M and N are constants and m is an exponent, all varying largely with particle shape. For example, for natural sediment grains, M = 32, N = 1 and m = 2/3, as proposed by Cheng [15]. For spherical particles, it can be shown that by taking M = 24, N = 0.4 and m = 0.6, Eq. (5) provides a good representation of classical data [16]. By noting that $C_D = (4/3)(\Delta g D / w^2)$ and $Re = wD / \nu$,

$$\frac{3}{4} C_D \mathrm{Re}^2 = \frac{\Delta g D^3}{\nu^2} = D_*^3 \tag{6}$$

where

$$D_* = \left(\frac{\Delta g}{\nu^2}\right)^{1/3} D \tag{7}$$

is the dimensionless diameter, Eq. (5) can be rewritten to be

$$C_D = \frac{4}{3} D_*^3 \left(\sqrt{\frac{1}{4}\left(\frac{M}{N}\right)^{2m} + \left(\frac{4}{3}\right)^m \frac{D_*^{3m}}{N^m}} - \frac{1}{2}\left(\frac{M}{N}\right)^m \right)^{-2/m}. \tag{8}$$

It is noted that D_* describes the gravitational force in comparison to the viscous force, and $D_* = A_r^{1/3}$ where A_r is the Archimedes number [17]. Different from Re, D_* is independent of w. Therefore, using Eq. (8), C_D can be calculated for a grain of particular shape with known values of M, N, m, Δ, D and ν. In the subsequent analysis, Eq. (8) will be used to develop a pseudo-fluid model. However, it is noted that Δ is not a parameter physically applicable for a packed bed, but it can be expressed as a function of the hydraulic gradient and porosity, as shown later.

2.1. Drag exerting on a grain in sediment bed

Consider a sediment bed of uniform grains. It is assumed that the bed in the streamwise direction is long enough so that the flow through the bed can be considered fully developed. Two scenarios are compared here, as sketched in Fig. 1. The first is a sediment bed applied with an upward flow, of which the hydraulic gradient is S, the cross-sectional average velocity is V, and the average velocity through the pores is V_s (= V /

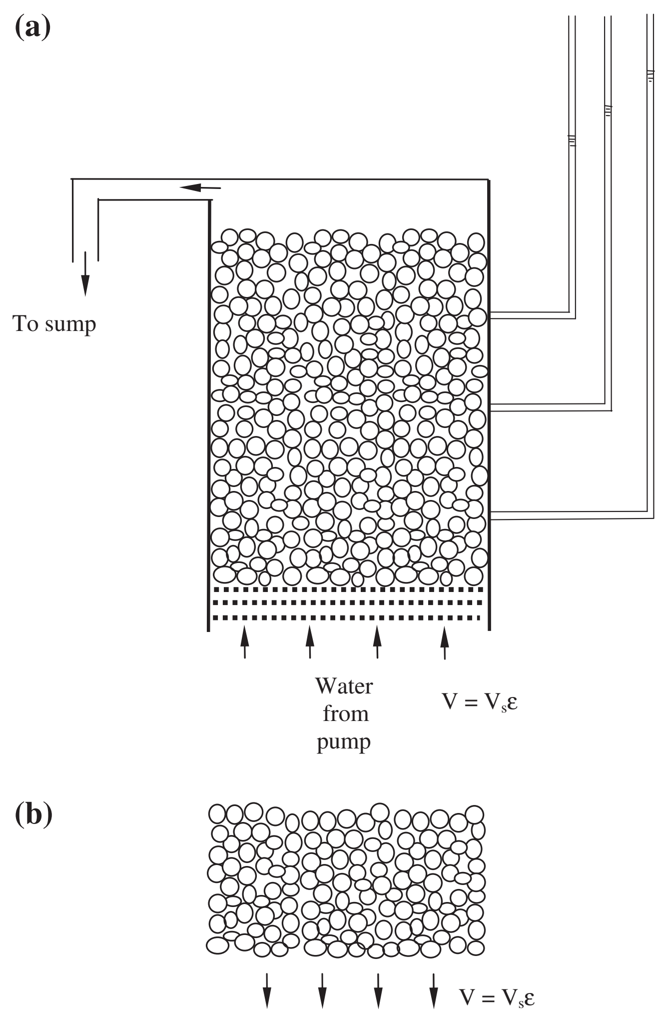

Fig. 1. Two scenarios: (a) water seeping through gravel bed; (b) gravel bed settling in still water.