Powder Technology 260 (2014) 42–51

Contents lists available at ScienceDirect

Powder Technology

journal homepage: www.elsevier.com/locate/powtec

One-dimensional modeling of concentration distribution in pipe flow of combined-load slurry

Václav Matoušek *, Jan Krupička

Institute of Hydrodynamics, Academy of Sciences of Czech Republic, Pod Patankou 30/5, 166 12 Prague, Czech Republic

ARTICLE INFO

Article history:
Received 4 December 2013
Received in revised form 23 March 2014
Accepted 30 March 2014
Available online 4 April 2014

Keywords:
Hydraulic transport
Sediment transport
Two-phase flow
Experiment
Bed shear

ABSTRACT

A one-dimensional profile of solids concentration is modeled in a cross section of partially stratified flow of slurry in a pipe. In the flow, a certain proportion of solids is transported as contact load and occupies a sliding bed and a transport layer above the bed. The rest of solid particles is transported as suspended load within and above the transport layer. A model based on a vertical distribution of shear stress is proposed to predict a concentration profile in a cross section of such flow. Besides the concentration profile, the model predicts the thickness and velocity of the bed. Furthermore, the model determines a vertical position of the top of the transport layer and a position of the hydrodynamic axis of the flow. Model predictions show a satisfactory match with new experimentally determined profiles collected in slurry flows of four different fractions of glass beads in a 100-mm pipe of our laboratory loop.

1. Introduction

Predictive models for a distribution of solid particles in slurry flowing through a pressurized pipe should be based on a mathematical description of particle support mechanisms. Basically, there are two different mechanisms that can support particles in flowing slurry: the interaction of a particle with turbulent eddies of the flowing carrier and the interaction of a particle with other particles through permanent contacts (grains in a bed) or sporadic contacts. Hence, particles that do not contribute to the bed can be dispersed throughout the slurry flow either by a diffusive action of turbulent eddies or by collisions between particles traveling at different velocities. The different mechanisms tend to produce rather different shapes of profiles of local solid concentration across a discharge area of slurry flow. Quite often, both mechanisms are active in the flow, each supporting a certain proportion of transported particles. Modeling of concentration profiles in such flows is quite difficult as a mathematical description of different particle-support mechanisms and their mutual interactions is still rather crude and unreliable. Recently, first attempts have been made to employ CFD to model concentration profiles of heterogeneous slurry flows in pipes [1–3]. Nevertheless, practice still requires engineering models that are simpler and thus one-dimensional (1-D). Such models predict a distribution of concentration along the vertical axis of a flow cross section leaving a value of the local concentration constant in the transverse direction. A typical method for measurement of concentration profiles is vertical scanning of a pipe cross section by a gamma-ray beam and this method produces the same type of *c*-profiles.

A typical 1-D model is a turbulent-diffusion model of the Schmidt–Rouse type. It exists in various modifications for various slurry-flow conditions in pipes, recently proposed versions are in [4,5]. A typical modification takes high average concentration of solids in the slurry into account through implementing the hindered settling effect [6–9]. Some model modifications introduce an effect of a broad particle size distribution of a transported solid fraction [10,11]. Furthermore, a key parameter of the turbulent-diffusion model, the particle diffusion coefficient, is modeled in different ways in different versions of the diffusion model (e.g. a survey in [12]). Kaushal and Tomita [5] upgraded their 2002-version of the turbulent-diffusion model to improve its accuracy in flows of slurries of narrow graded solids. They stated that their model was appropriate to different flow patterns including flow regions with a zero *c*-gradient (e.g. stationary- and sliding beds). Our recent analysis of *c*-profiles in stratified flows suggested [13] that modeling of small *c*-gradients observed in stratified flows using a turbulent-diffusion model in which high values of the dimensionless particle diffusivity took care of the small *c*-gradient did not seem to be appropriate.

1-D modeling of flows carrying particles supported by mutual collisions is associated with a concept of a transport layer in which a *c*-gradient is virtually constant [14,15]. Our one-dimensional stress-distribution-based model (1-D SDM) for flow of contact-load slurry [13] employs the concept as well. The proposed model predicts a shape of a *c*-profile by determining vertical positions of the top of the

Abbreviations: 1-D SDM, one-dimensional stress distribution based model; CSA, cross-sectional area; SLB, sliding bed; TL, transport layer.

* Corresponding author. Tel.: +420 224 354 673; fax: +420 224 354 793.
E-mail addresses: matousek@ih.cas.cz (V. Matoušek), krupicka@ih.cas.cz (J. Krupička).

http://dx.doi.org/10.1016/j.powtec.2014.03.070

sliding bed and of the top of the transport layer provided that all particles are transported as contact load.

Hybrid modeling is required for slurry flows carrying combined loads (both contact load and suspended load are present) with as a result a model that takes both solid support mechanisms into account [7,16]. Available predictive models for c-profiles in combined-load flows are inadequate, particularly in predicting a sharp change in a c-gradient at the interface between a sliding bed and flow above the bed [13]. A model is required that considers the existence of a zero c-gradient within a sliding bed (the feature that the existing models are not able to satisfy properly) and non-zero local c-gradients in contact-load dominated transport layer above the bed. Such a model must be validated using experimentally determined c-profiles in slurry pipes.

There is a lack of suitable experimental data for calibration and validation of models dealing with slurry flow carrying a combined load. An experimental database of c-profiles and associated flow quantities in pressurized pipes contains more data for fine pseudo-homogeneous flows, e.g. [8,11,17–19], and coarse strongly stratified flows, e.g. [20,21], than data for partially-stratified flows of medium-to-coarse slurries (typically particles of quartz density and sizes between 200 and 1000 μm). The literature with experimental c-profiles in medium-to-coarse slurries is limited to a few resources [7,8,11,17,22–24]. Recently, we extended the database with c-profiles for four fractions of medium-to-coarse ballotini [13,25,26], including the data for flows with sliding beds.

To summarize, experimental data for slurry flow of combined load with sliding bed are scarce and the most common predictive models for c-profiles do not take the stratified nature of the combined-load flow sufficiently into account. The objective of this work is to extend the applicability of the 1-D SDM model from flow of contact load to flow of combined load and to validate the proposed model using newly collected experimental c-profiles.

2. Theoretical considerations

An evaluation of a 1-D distribution of applied- and resisting shear stresses in a cross section enables determination of vertical positions, y, of interfaces of characteristic layers in a stratified flow of slurry. The stress distribution is further used to establish a distribution of the volumetric concentration of grains, c, along y, i.e. to determine a concentration profile of $c(y)$, in the layered flow pattern.

2.1. Shear stress distribution in stratified flow carrying contact load

Before we apply the stress-distribution concept to flow through a circular pipe, let us demonstrate the concept principles on a simpler geometry of two parallel plates. We demonstrate a relation between a distribution of shear stresses and a distribution of grain concentration c on an example of stratified flow of contact load composed of a bed (a constant value of c at each vertical position y) and a transport layer adjacent to the top of the bed (a linear distribution of c across the layer with the zero value at the top of the layer), see Fig. 1a. In Fig. 1a, c_b is the local volumetric concentration of solids in the bed, D is the vertical distance between the lower boundary and the upper boundary of the flow, y_b is the vertical position of the top of the bed, H_{TL} is the thickness of the transport layer and $R_{x,w}$ is the vertical position of the longitudinal hydrodynamic axis of the flow, i.e. the vertical position of the zero shear stress, $\tau(R_{x,w}) = 0$. The distribution of c deforms a profile of fluid velocity, u_f, and shifts the position of the hydrodynamic axis ($y = R_{x,w}$) towards the upper boundary of the flow.

At each vertical position y (the origin of y is at the bottom of a flow cross section), the general stress-balance equation applies,

$$-\frac{dp}{dx} = \frac{\tau}{R_{x,w} - y} \quad (1)$$

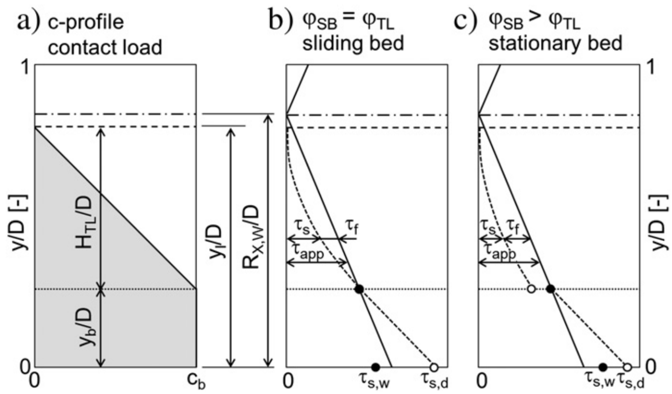

Fig. 1. Schematic distribution of volumetric concentration and shear stress along vertical axis of flow cross section in stratified flow carrying contact load. Legend: black circles at y_b/D are τ_s for $\tan\varphi_{SLB}$; blank circle at y_b/D are τ_s for $\tan\varphi_{TL}$.

in which dp/dx = pressure gradient due to friction, and τ = total resisting shear stress at vertical position y. The resisting stress τ balances the stress applied by the flow at each y-position, $\tau_{app} = -\frac{dp}{dx}\cdot\left(R_{x,w} - y\right)$. The pressure differential is considered independent of a vertical position, $\frac{dp}{dx}(y) = \text{const}$, and hence the distribution of both the applied- and resisting stresses is linear along the y-axis with a zero value at $y = R_{x,w}$. The slope of the distribution is

$$\frac{d\tau_{app}}{dy} = \frac{dp}{dx}. \quad (2)$$

In solid–liquid flow with contact load, $\tau(y) = \tau_s(y) + \tau_f(y)$, where $\tau_s(y)$ is the local solid shear stress and $\tau_f(y)$ is the local fluid shear stress (Fig. 1b). The solid shear stress $\tau_s(y)$ is a product of the submerged weight of contact-load grains above the position y and it is related to the normal solid stress

$$\sigma_s(y) = \int_y^D (\rho_s - \rho_f)\cdot g\cdot c\cdot dy \quad (3)$$

(ρ_s is the density of solids, ρ_f is the density of fluid, g is the acceleration of gravity) through the coefficient of internal friction of grains $\tan\varphi = \tau_s/\sigma_s$.

The solid stress increases with the depth below the top of the transport layer towards the lower boundary of flow. The normal stress at the lower wall is

$$\sigma_{s,w} = \sigma_s(0) = (\rho_s - \rho_f)\cdot g\cdot C_{vi}\cdot D \quad (4)$$

where C_{vi} = average volumetric concentration of grains in cross-sectional vertical D. The corresponding solid shear stress at the wall, $\tau_{s,w}$, is related to $\sigma_{s,w}$ through the coefficient of mechanical friction between granular bed and wall, μ_s.

Throughout the transport layer (TL), the shape of the shear-stress-distribution curve causes the value of the τ_s-gradient to increase with the depth (Fig. 1b) provided that a value of the coefficient of internal friction of grains in the transport layer $\tan\varphi_{TL}$ is considered constant, i.e. independent of y. At the interface between the transport layer and the sliding bed (SLB), $y = y_b$, the minimum value of the τ_s-gradient is reached. Its value is

$$\left(\frac{d\tau_s}{dy}\right)_{min} = -c_b\cdot\tan\varphi_{SLB}\cdot(\rho_s - \rho_f)\cdot g \quad (5)$$