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High accuracy analysis for motion of a spherical particle in plane Couette fluid flow by Multi-step Differential Transformation Method

M. Hatami, M. Sheikholeslami, G. Domairry *

Department of Mechanical Engineering, Babol University of Technology, Babol, IRAN

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ABSTRACT

In this study, coupled equations of particle's motion in Couette fluid flow are solved by Multi-step Differential Transformation Method (Ms-DTM) considering the rotation and shear effects on lift force and neglecting gravity. The precious achievement of the present work is introducing a new, fast and efficient analytical technique for spherical particles in plane Couette fluid flow over the previous numerical and analytical results in the literature. Also, obtained values are compared with numerical solution and HPM–Pade which recently are done by Torabi et al. It is shown that presented method gives approximations with a high degree of accuracy and least computational effort for studying particle motion in Couette fluid flow without the need for any linearization, discretization or perturbation against the previous work. Also, the acceleration profiles of the particle are presented and positions of the particle in each 1 s time step are depicted graphically in the current study.

1. Introduction

The numerical solutions of equation derived from the balance equation of external forces applied to a particle in interaction with drag, virtual mass force, the Basset force, lift force, etc. have been broadly considered in literature; albeit an exact analytical expression that gives a better understanding of the physical behavior of the sphere-fluid combination and the accelerated motion of the sphere is still worth studying. Knowing that, an analytical solution is usually the more preferred and convenient method in engineering area because of less computational work as well as high accuracy. The cases that particles come into contact with a solid or fluid boundary and obtaining the drag coefficient in order to predict the free falling velocity of particles have been an immense interest for many researchers. Various processes such as filtration, combustion, air and water pollution, coal transport and cleaning and micro contamination control are involved the particle transportation and deposition. Many of the previous analytical studies are constructed on particle sedimentation in Newtonian media which some of them are introduced in the following paragraph.

Jalaal et al. [1] solved a spherical particle's motion in Couette flow using Homotopy perturbation method (HPM [2]) and got comparable results to numerical ones. The unsteady rolling motion of a spherical particle restricted to a tube was studied analytically by Jalaal and Ganji [3]. They obtained an exact solution of particle velocity and acceleration motion under some practical conditions through applying HPM. Jalaal and Ganji [4] proposed an analytical solution for the acceleration motion of a spherical particle rolling down an inclined boundary with drag coefficient which is correlated linearly to *Re* in a specific range using HPM. They studied various inclination angles and observed that settling velocity, acceleration duration and displacement are proportional to the amount of inclination angle while for a constant inclination angle, settling velocity and acceleration duration are decreased by increasing the fluid viscosity. Torabi and Yaghoobi [5], investigated the better performance of a combination of He's polynomials and the diagonal Padé approximants rather than HPM. They showed that by calculating the approximate solution of the acceleration motion of a single spherical particle moving in a continuous fluid phase, HPM–Padé is a useful method. In a later study, they obtained the acceleration trajectory of a non-spherical particle moving in a continuous fluid phase using VIM–Padé approximants with acceptable accuracy compared to numerical results [6]. Recently, Ghasemi et al. [7] discussed about the convergency and accuracy of VIM (Variational Iteration Method) and ADM (Adomian Decomposition Method) for solving the motion of a spherical particle in Couette fluid flow. Also, Hamidi et al. [8] and Torabi et al. [9] confirmed that HPM–Padé is more accurate than HPM [1] for solving this kind of problem. All of the above researches for solving the particle motion in plane coquette fluid flow have some shortcomings such as low accuracy, needing linearization, discretization or perturbation, and high processing time, so the requirement for an analytical method which overcomes these drawbacks is completely

* Corresponding author. Tel./fax: +98 111 3234205.
E-mail addresses: m.hatami2010@gmail.com (M. Hatami), m.sheikholeslami@stu.nit.ac.ir (M. Sheikholeslami), G.Domairry@gmail.com (G. Domairry).

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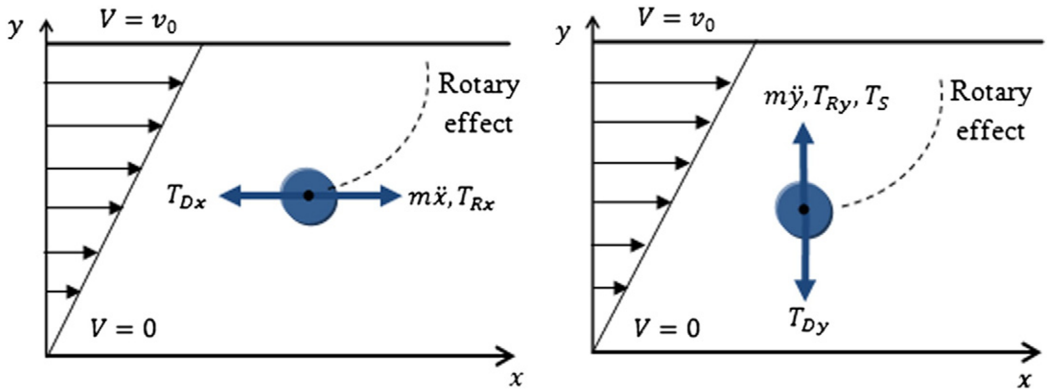

Fig. 1. Schematic view of exerted forces on a spherical particle in Couette fluid flow [7].

evident. Recently several authors published their papers about droplet [10–16] and bubble dynamics [17–20].

Multi-step Differential Transformation Method (Ms-DTM) which is a modified form of classical DTM introduced by Zhou [21], can be a suitable series solution for this kind of engineering and physical problems. This method is newly used in some other engineering problems [22–24]. Some other modifications on DTM such as GDTM [25] (Generalized DTM) can be found in literature. Other series solution method is used in many problems. For example Hatami and Ganji [26] and Sheikholeslami et al. [27] used Least Square Method for solving the heat transfer in circular porous fins and porous channel respectively. Haji-Sheikh et al. [28,29] studied heat transfer in porous passage through weighted residual methods and series solution. Khan et al. [30,31] used series solutions for flow analysis in porous slider and heat transfer of MHD non-Newtonian fluids, respectively. Also new kinds of numerical and semi analytical methods were successfully applied in different scientific problems [32–70].

Motivated by the above mentioned works, the main objective of the present research paper is to introduce the Ms-DTM as a highly accurate and efficient method on the 2D motion of a spherical particle in plane Couette fluid flow which overcomes the shortcomings of the previous studies. In contrast to the results of the previous studies, this method is very fast, accurate and for all constant values in coupled equations of the problem is in excellent agreement with numerical method.

2. Statement of problem

The equation of motion for spherical particle lift effect in a Couette flow has been studied by Steverding [71]. In that study the lift force of particle was assumed due to the mere rotation of particle, while later Saffman [72] proved that the contribution of shear effect in particle's motion is not less important than the former one. Vander Werff proposed a more comprehensive equation of motion of spherical particle in the Couette flow which covers shortfalls of previous studies [73].

The Vander Werff model for particle motion in the Couette flow is adopted in this study while the positive direction rotation of particle is clockwise and combined the effects of inertia, gravity and buoyancy forces are assumed to be negligible [74]. So, the inertia force in the left hand side of force balance equation is the product of spherical particle mass by its acceleration $(x, y, 0)$:

$$T = \frac{4\pi a^3}{3}\rho_s \dot{V} = \frac{4\pi a^3}{3}\rho_s\left(\ddot{x}, \ddot{y}, 0\right) \tag{1}$$

where a, ρ and V denote the radius, density and velocity of spherical particle, respectively.$\dot{V}$is the first derivative of particle's velocity and x and y are the second derivatives of particle motion in horizontal and vertical directions with respect to time.

In order to calculate the drag force, the velocities of the spherical particle are considered small adequately so that the Stokes law can be governed:

$$T_{Dx} = 6\pi\mu a V_{rx} = 6\pi\mu a(\dot{x} - \alpha y) \tag{2a}$$

$$T_{Dy} = 6\pi\mu a V_{ry} = 6\pi\mu a\dot{y} \tag{2b}$$

while μ signifies the viscosity of fluid.

Balancing the inertial with drag and lift, the equation of motion is obtained as:

$$T_{Rx} = \frac{1}{2}\pi a^3 \rho\alpha\dot{y} \tag{3a}$$

$$T_{Ry} = \frac{1}{2}\pi a^3 \rho\alpha(\alpha y - \dot{x}) \tag{3b}$$

$$T_{Sx} = 0 \tag{4a}$$

$$T_{Sy} = 6.46 a^2 \rho\alpha^{1/2}\nu^{1/2}(\alpha y - \dot{x}) \tag{4b}$$

where v is the dynamic viscosity, $(\frac{\mu}{\rho})$, and α is defined as the positive proportionality constant.

An illustration of the spherical particle in the plane Couette fluid flow and applied forces on particle is shown in Fig. 1. The mass of particle is assumed to be in the center of sphere and the forces caused from the rotation and shear fields and their interactions on drag and lift forces of particle are illustrated in Fig. 1.a and .b respectively. By forming the forces balance equation of the inertia force to the drag and lift forces; the equations of motion for the particle are driven as:

$$T = T_R + T_S - T_D. \tag{5}$$

Eventually, by substituting the Eqs. (2a), (2b) and (4a), (4b) into Eq. (5) the system of equation of motion for spherical particle in the plane Couette flow yields:

$$\begin{cases} \frac{4\pi a^3}{3}\rho_s\ddot{x} = \frac{1}{2}\pi a^3\rho\alpha\dot{y} - 6\pi\mu a(\dot{x} - \alpha y) \\ \frac{4\pi a^3}{3}\rho_s\ddot{y} = \left(\frac{1}{2}\pi a^3\rho\alpha + 6.46 a^2\rho\alpha^{1/2}\nu^{1/2}\right)(\alpha y - \dot{x}) - 6\pi\mu a\dot{y}. \end{cases} \tag{6}$$