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Approximate analytical solution of squeezing unsteady nanofluid flow

A. Dib [a,*], A. Haiahem [b], B. Bou-said [c]

[a] Université 20 Août 1955, B.P. 26, Route d'El Hadaiek, Skikda 21000, Algeria
[b] Laboratory of Industrial Mechanics, University of Annaba, B.O. 12, 23000, Algeria
[c] Université de Lyon, CNRS INSA-Lyon, LaMCoS, UMR5259, F-69621, France

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ABSTRACT

In this paper, the Duan–Rach Approach (DRA) was used to obtain an approximate analytical solution of squeezing unsteady nanofluid flow. An approximate analytical solution can be obtained for a velocity and a temperature profile. This method modifies the standard Adomian Decomposition Method (ADM) by evaluating the inverse operators at the boundary conditions directly. The obtained results show a good agreement with numerical method (fourth order Runge–Kutta algorithm). The algorithm derived from this approach can be easily implemented.

1. Introduction

Squeezing unsteady viscous flow between two parallel plates is an interesting topic of research because it occurs in many industrial applications which include polymer processing, compression, injection modeling, lubricant system, transient loading of mechanical components and the squeezed films in power transmission, food processing and cooling water. It is well known that Choi [1] was the first to introduce the term nanofluid that represents the fluid in which nano-scale particles are suspended in the base fluid with low thermal conductivity such as water, ethylene glycol, and oil. An excellent articles on this field can be found in [2–6] and a recent book by Das et al. [7]. Governing equations arising from flow between moving plates should be solved numerically or analytically. Nanofluids are fluids with suspensions of metals, oxides, carbides or carbon nanotubes in a base fluid. Pantzali et al. [8,9] studied the effect of the use of a nanofluid in a miniature plate heat exchanger (PHE) with modulated surface both experimentally and numerically. They found that the considered nanofluids (CuO–water) can be a promising solution towards designing efficient heat exchanging systems, especially when the total volume of the equipment is the main issue. Mahian et al. [10] performed an analytical study for the flow and heat transfer of TiO_2/water nanofluid in a vertical annulus in presence of magnetohydrodynamic field. Wen and Ding [11] found that dispersion of Al_2O_3 nanoparticles in water can result in significant enhancement of convective heat transfer.

Bachok et al. [12] solved the nonlinear differential equations numerically by a shooting method; they found that dual solutions exist when the plate and the free stream move in the opposite directions. The analytical methods resolve a many problems in engineering field. Torabi and Yaghoobi [13] evaluated the velocity of a vertically falling spherical particle by HPM–Padé approximant. They found that this method can achieve more suitable results in comparison to homotopy perturbation method (HPM). Hamidi et al. [14] used the same method to approximate the motion of spherical solid particle in plane coquette fluid flow. In another work, Torabi et al. [15] associate Boubaker polynomials expansion scheme (BPES) with the HPM–Padé method to find the solution of a spherical particle in a plane coquette Newtonian fluid flow. Yaghoobi and Torabi [16] found an analytical solution for falling non-spherical particle by differential transformation method (DTM). In order to ameliorate the accuracy of the variational iteration method (VIM), Yaghoobi and Torabi [17] associate Padé approximant and VIM. Hashmi et al. [18] studied the squeezing flow of nanofluid in presence of the magnetic field. The solution was obtained analytically by homotopy analysis method (HAM).

Fatoorehchi and Abolghasemi [19] have proposed a very interesting Matlab code to calculate the Adomian polynomials. In recent papers, Fatoorehchi and Abolghasemi [20] found a new analytical approach for finding the roots to the minimum-reflux-ratio Underwood equation by using the ADM. The proposed scheme was equipped with a nonlinear convergence accelerator (Shanks transform). In another work, Fatoorehchi et al. [21] resolved by the same proposed method the Hankinson–Thomas–Phillips correlation for the prediction of the natural gas compressibility. Qin and Sun [22] solved the moving boundary problem by the ADM. Duan [23] proposed a recurrence technique for

* Corresponding author.
E-mail addresses: amar_dib2000@yahoo.fr (A. Dib), a.haiahem@yahoo.fr (A. Haiahem), b.bousaid@lmcos.fr (B. Bou-said).

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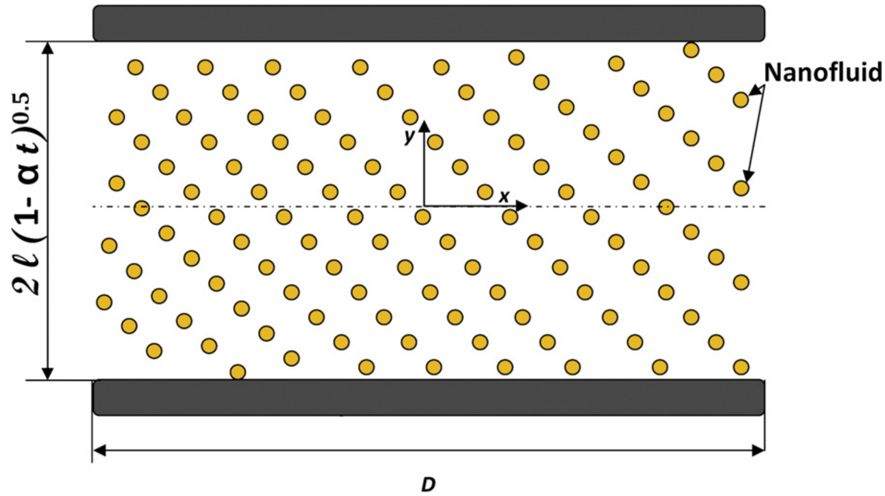

Fig. 1. Geometry of problem.

calculating Adomian polynomials to obtain a large domain of convergence. Rach [24] developed a theoretical background of the Adomian Decomposition Method under the auspices of the Cauchy–Kovalevskaya theorem of the existence and uniqueness for solution of differential equations. Sheikholeslami et al. [25] investigated the heat transfer characteristics in squeezed flow by using the Adomian Decomposition Method (ADM). They used a Maxwell-Garnett (MG) model and a Brinkman model, respectively. They found that for the case in which two plates are moving together, the Nusselt number increases with the increase of the nanoparticle volume fraction and Eckert number while it decreases with the growth of the squeeze number. In another paper, Sheikholeslami [26] used homotopy perturbation method to obtain the analytic solution under the same hypothesis. Domairry and Hatami [27] investigated this problem by using a differential transformation method (DTM–Padé); they found that Padé's order [6,6] can be an exact solution.

All of the analytical methods used by the authors are not purely analytic method. The standard Adomian Decomposition Method used by Sheikholeslami and Ganji [25] turns the boundary value problem (BVP) into an initial value problem (IVP). For achieving that, we need to find the unknown initial values of the problem ($f'(0)$, $f'''(0)$, $\theta(0)$) by numerical methods. The first inconvenient is that the final solution depends on the accuracy of the initial values determined by numerical method. Second inconvenient is that we need to perform all the calculation at any variation of the parameters of the flow. Duan and Rach [28] have presented a new modification of the ADM to solve a wide class of multi-order and multi-point nonlinear boundary value problems (BVPs). In this study, we succeed to overcome all those inconvenient by using a Duan–Rach Approach (DRA) to find an approximate analytical solution.

2. Problem description

The unsteady flow and heat transfer in two-dimensional squeezing nanofluid between two infinite parallel plates is considered in this work (Fig. 1). The thermophysical properties of the nanofluid are given in Table 1. The hypothesis of the problem can be found in more detail in Ref. [25]. The governing equations are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\rho_{nf}\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu_{nf}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \tag{2}$$

$$\rho_{nf}\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu_{nf}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \tag{3}$$

$$\begin{aligned}\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{nf}}{\left(\rho C_p\right)_{nf}}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)\\+\frac{\mu_{nf}}{\left(\rho C_p\right)_{nf}}\left(4\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)^2\right),\end{aligned} \tag{4}$$

where u and v are the velocities in x and y directions, respectively. The effective density (ρ_{nf}), the effective dynamic viscosity (μ_{nf}), the effective heat capacity $(\rho C_p)_{nf}$ and the effective thermal conductivity (k_{nf}) of the nanofluid are defined as [29]:

$$\begin{aligned}\left(\rho C_p\right)_{nf}&=(1-\phi)\left(\rho C_p\right)_f+\phi\left(\rho C_p\right)_s,\\ \rho_{nf}&=(1-\phi)\rho_f+\phi\rho_s,\\ \mu_{nf}&=\frac{\mu_f}{(1-\phi)^{2.5}},\quad \text{(Brinkman)}\\ \frac{K_{nf}}{K_f}&=\frac{K_s+2K_f-2\phi\left(K_f-K_s\right)}{K_s+2K_f+2\phi\left(K_f-K_s\right)}\text{(Maxwell–Garnett)},\end{aligned} \tag{5}$$

subject to the following boundary conditions:

$$\begin{aligned}&v=v_w=\frac{dh}{dt},\quad T=T_H\quad \text{at}\quad y=h(t),\\ &v=\frac{\partial u}{\partial y}=\frac{\partial T}{\partial y}=0\qquad \text{at}\quad y=0.\end{aligned} \tag{6}$$

Table 1
Thermophysical properties of fluid and nanoparticles [31].

Physical properties	Fluid phase (water)	Copper (Cu)	Alumina (Al_2O_3)	Titanium oxide (TiO_2)
C_p (j/kg K)	4179	385	765	686.2
ρ (kg/m^3)	997.1	8933	3970	4250
K (W/mK)	0.613	400	40	8.9538