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# Non-linear behaviours in complex fluid dynamics via non-differentiability. Separation control of the solid components from heterogeneous mixtures

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ABSTRACT

In the framework of the Scale Relativity Theory, non-linear effects in complex fluids and, implicitly, the separation control of the solid components from heterogeneous mixtures are analysed. Assuming that the movements of the complex fluid entities occur on continuous but non-differentiable curves, the specific momentum and the local energy density conservation law equations are obtained. For potential movements at a fractal scale, as well as for non-potential ones at a differentiable scale, the non-differentiable hydrodynamic model is established. In such context, for different curve motions at various resolution scales, the non-differentiable hydrodynamic model is reduced to either a quantum hydrodynamic model, a standard hydrodynamic model, a random walk model, or a Stokes model. By numerical simulations using the non-differentiable hydrodynamic equations and the internal energy density conservation law with adequate initial and boundary conditions, some non-linear effects are obtained. Moreover, eliminating the time between the viscosity stress tension type and temperature field, for various given positions, thermal hysteresis type effects can be obtained. For each of the models mentioned above, the separation control processes are discussed.



## 1. Introduction

The natural functionalities of a fluid entail the dynamics of fluid–solid mixture flows. Furthermore, the same dynamics generate many applications in the industry: the co-current down-flow circulating fluidised bed reactors [1], the optimisation of air de-pollution installations [2], the sedimentation of solid particles in a turbulent flow in horizontal channels [3], the mixture separation regimes of solid particles [4–7], etc.

Theoretical models have investigated the influences of mixture parameters on velocity flows (e.g., the properties of solid particles [8–11]) or the distribution of solid particles due to the rotational regimes of these flows in a fluid [12–14]. An interesting review of the numerical simulations of fluid–solid mixture flows was published by Zhang et al. [15]. According to the usual concepts [16–18], all of these theoretical models assume that the dynamics of both the fluid and the solid particles in fluid–solid mixtures occur on continuous and differentiable curves [16,17], so such dynamics can be described by continuous and differentiable functions (e.g., density, velocity or temperature fields). These functions are exclusively dependent on the spatial coordinates and time. Usually, the computational fluid dynamic models start from one, or a set of differential equations, discretised by means of finite element method, the increased precision of the model implying powerful hardware systems and long time of operation.

However, in reality, a fluid–solid mixture flow proves to be much more complex. Therefore, the above simplifications cannot explain all of the aspects of the flow dynamics. Thus, a new mathematical formalism is needed that takes into account the complexity of the dynamics of fluid–solid mixture flows.

The first step in developing new mathematical formalism was made in [19]. By analysing the dynamics that was induced by different types of solid particle–real fluid interactions, which generated a boundary layer, we showed that, after imposing adequate initial and boundary

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conditions, the dynamic equations of the fluid–solid mixtures (the Prandtl and continuity equations for plane symmetry) generate local soliton, kink and soliton–kink nonlinear solutions for the velocity field of the fluid. Local velocity gradients, which act at the boundary layer limit, then induce rotational movements in a fluid, which are transferred to an external solid particle due to adherence and fluid viscosity. Thus, the external solid particle jumps from one streamline to another.

Considering the Scale Relativity Theory (SRT) [20–22], such behaviour in Euclidian space (the movement along a streamline followed by a jump from one streamline to another) is replaced by movements on continuous but non-differentiable curves (fractal curves) in a fractal space [20–27]. Consequently, the Euclidian dynamics of fluid–solid mixture with constraints (interactions) are substituted by the fractal dynamics of complex fluid that is free of any constraints. Complex fluid entities move along continuous but non-differentiable curves (fractal curves) that have a double identity; these curves are both the geodesics of a fractal space and the streamlines of the complex fluid. The dynamics of complex fluid can be described using fractal quantities (fractal density, fractal momentum, fractal energy, etc.), i.e., functions that depend on the spatial coordinates, time and resolution scales. In such a conjecture, the complex fluid has specific properties, such as a hysteretic one (memory) [28,29].

Considering the presented arguments, using the SRT mathematical formalism, we showed in [30], that the entities of a complex fluid move along the fractal curves that are described by the Navier–Stokes type equations of a complex velocity field with complex structure coefficients (imaginary coefficient of the viscosity type, etc.). In this approximation of the movement, which was termed by us the convective–dissipative approximation of the movement, the local self-acceleration, self-convective and self-dissipative effects are in equilibrium at any point of any movement path. These trajectories are simultaneously assimilated, even with fractal space geodesics or with the streamlines of the complex fluid. The complex velocity field that is correlated with the resolution scales is mathematically translated into the separation of the real part from the imaginary part, which induces the hydrodynamic fractal model (HFM). The HFM contains density and momentum conservation laws, and these laws are dependent on the resolution scales. The chaoticity (fractality) of the movement trajectories is assimilated into a fractal potential, which is only dependent on the imaginary part of the velocity field, with its gradient "functioning" as a force in the momentum conservation law. For a particular type of the fractal potential, the nonlinear numerical solutions of the HFM, such as soliton–antisoliton and soliton–soliton package type solutions, specify the "non-differentiable" behaviour of the movement trajectories. Furthermore, independent of the complex fluid flow type, the real part of the complex velocity generates a flow (a vortex intensity), while the imaginary part indirectly induces a generalised lift force.

The present paper gives extended results from [30], taking into account that the non-linear effects play an important role in the flow process of complex fluid. Such a flow process frequently occurs in multi-scale type structures, presenting a temporal diffusion scale, a temporal convection scale, a temporal heterogeneous reaction, etc.; e.g., in fluidised bed material systems [31]. The dynamics of multi-scale-type structures imply the mathematical formalism of the SRT. Moreover, in our opinion, the non-linear effects induce the specific mechanisms of the mixture separation regimes of solid particles in complex fluid flows. Till now, the problem of the segregation of solid particles from the fluid–solid mixtures has been studied using Eulerian–Lagrangian models and others [32–34]. Let us note that the Eulerian–Eulerian approach is the most widely used for fluidised bed applications.

The present paper is structured as follows: Section 2 — hallmarks of non-differentiability and conservation laws; Section 3 — non-differentiable hydrodynamic model and its correspondences with standard models; Section 4 — non-linear effects in complex fluids through numerical simulations and Section 5 — conclusions.

## 2. Hallmarks of non-differentiability. Conservation laws

For developing our theoretical model, we take into account that, in complex fluid, deterministic chaos arises in association with spatio-temporal structure emergence. For temporal scales that are large with respect to the inverse of the highest Lyapunov exponent, the deterministic trajectories can be replaced by collections of potential trajectories and the concept of definite positions by that of the probability density. This concept was introduced in the framework of the SRT [21,22], which states that the particle movements take place on continuous but non-differentiable curves (fractal curves). Subsequently, all physical phenomena become dependent not only on the spatio-temporal coordinates but also on the spatio-temporal scales. Thus, the non-differentiability becomes a fundamental characteristic of the complex fluid dynamics.

The main consequences of non-differentiability are the following [21,22,35–39]:

i) A continuous and non-differentiable curve (or almost nowhere differentiable) is explicitly scale dependent. Moreover, its length tends to infinity, when the scale interval tends to zero. Consequently, according to Mandelbrot's concept, continuous and non-differentiable space will be a fractal space [25]. Thus, there are infinite fractal curves (geodesics) relating to any couple of points (or starting from any point), and this is valid for all resolution scales. Then, the entities of the complex fluids may be reduced to and identified with their trajectories, so that the complex fluids will behave as a special interaction-less fluid (fractal fluid);
ii) Physical quantities will be expressed through fractal functions, namely those functions depending on space–time coordinates as well as on resolution scale. The invariance of the physical quantities in relation with the resolution scale generates special types of transformations that are called *resolution scale transformations*. Let us now explain the above statements through an example in nano-fluids. We can choose the fractal normalised heat flux–fractal normalised temperature characteristic in the form:

$$\overline{T} = \overline{J}\left(1 + \frac{\mu}{1 + \overline{J}^2}\right) \tag{1}$$

where $\overline{T}$ is the fractal normalised temperature, $\overline{J}$ is the fractal normalised heat flux and $\mu$ is a parameter depending on scale resolution. This relation induces thermal conduction bistability (see Fig. 1) as follows:

— the restriction $\mu \geq 8$ implies bistability;
— the value of $\mu$ sets the scale resolution through the thermal transfer regimes of the complex fluid;
— once $\mu$ is fixed (with $\mu \geq 8$), for values of the fractal normalised heat flux in the interval AB on the characteristic (see Fig. 1) the fractal normalised temperature can have two distinct stable values;
— thermal conduction bistability is associated with a negative differential thermal resistance (or thermal hysteresis) [40];
— since $\overline{T}$ and $\overline{J}$ are fractal functions (relation (1)) they can exhibit the property of self-similarity. Consequently, thermal conduction bistability in Fig. 1 can occur at any scale resolution (i.e. for different thermal transfer regimes of the complex fluid);