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# A 3-parameter model for packing density prediction of ternary mixes of spherical particles

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ABSTRACT

The conventional packing models are mostly based on the linear packing theory that for a mixture of several size classes of particles (each size class is a collection of mono-sized particles), the specific volume of the mixture is a linear function of the volumetric fractions of the individual size classes. However, deviations from this theory at certain volumetric fractions leading to significant reduction in packing density have been observed. The authors attribute such deviations to the newly identified wedging effect of the fine particles stuck at the gaps between the coarse particles. Herein, the 3-parameter packing model with the wedging effect incorporated is extended to ternary mixes of spherical particles. In the formulation, the linear packing theory is not followed in the sense that the specific volume is no longer assumed to be a linear function of the volumetric fractions. The extended model is validated by checking against the test results from this study and the literature.


## 1. Introduction

For granular materials made of solid particles, the packing characteristics have great influence on their properties, such as flowability, suspension rheology, porosity, permeability and strength, etc. [1–7]. Therefore, knowledge of particle packing is of prime importance in many industries involving granular materials. Among the various packing characteristics, the packing density (ratio of the solid volume of particles to the bulk volume of particles), which determines the void content and often governs the overall performance of the granular material, is the most widely used parameter for characterizing a particle system. Hence, it is useful to develop a particle packing model for predicting the packing density so that the various packing density related properties of the granular material can be analyzed and optimized.

Over the decades, a number of particle packing models have been developed to predict the packing density of granular materials [8–17]. In early 1930s, Westman and Hugill [18] and Furnas [19] initiated the packing modeling for binary mixes of mono-sized particles by studying the packing structures in the particle skeleton and analyzing the effects of the various packing structures on the packing density. Their models consider two limiting cases of the size ratio (ratio of the size of fine particles to the size of coarse particles). At a size ratio equal to unity (the fine and coarse particles have the same size), there is no structural effect that would affect the packing density. On the other hand, at a size ratio close to zero (the fine particles are very small compared to the coarse particles), there are two structural effects that would increase the packing density, namely: the filling effect of the fine particles filling into the voids between the coarse particles when the coarse particles are dominant, and the occupying effect of the coarse particles occupying solid volumes in place of porous bulk volumes of the fine particles when the fine particles are dominant.

Subsequently, Stovall et al. [20] introduced the so-called packing restrictions that would take place when the size ratio is neither equal to unity nor close to zero. These packing restrictions, or rather structural effects that would decrease the packing density, are namely: the loosening effect of the fine particles loosening the packing of the coarse particles while squeezing themselves into the voids between the coarse particles when the coarse particles are dominant; and the wall effect of the coarse particles disrupting the regular packing of the fine particles and thereby producing larger voids at the wall-like boundaries of the coarse particles when the fine particles are dominant. Both the loosening effect and the wall effect are dependent on the size ratio. They are separately taken into account by two parameters expressed as interaction functions of the size ratio.

Yu and Standish [21], Yu et al. [22], and de Larrard [23] derived their own interaction functions for the loosening effect parameter and the wall effect parameter in their models by curve fitting of the packing density results of binary mixes of spherical particles obtained from various sources. These models may be classified as the 2-parameter models as there are two parameters respectively accounting for the loosening and wall effects. To improve their applicability and accuracy, these models, originally developed for spherical and non-cohesive coarse particles, were refined for applications to non-spherical particles [22], particles subjected to different compactions during packing [23], very fine and cohesive particles [24] and mixtures containing both cohesive and non-cohesive particles [25].

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According to the 2-parameter models, the specific volume (ratio of the bulk volume of particles to the solid volume of particles, or in other words, the reciprocal of the packing density) of a mix of several size classes of particles, when a particular size class is dominant, is a linear function of the volumetric fractions of the individual size classes (each volumetric fraction is the solid volume of one particular size class of particles, expressed as a fraction of the total solid volume of all particles). This theory is called the linear packing theory [12,23]. Based on this theory, the specific volume may be determined as the maximum over a set of linear functions, each for the case of one particle size class being dominant. Hence, for a binary mix of two size classes, the specific volume is the maximum over two linear functions, one for the case of fine particles being dominant and the other for the case of coarse particles being dominant. These functions are plotted as straight lines in Fig. 1 to show how the specific volume varies. Note that there is a sharp corner at where the two straight lines intercept.

However, experimental results such as those obtained by de Larrard [23] and by Kwan et al. [26] revealed that the specific volume is not always a linear function of the volumetric fractions. For illustration, the results by Kwan et al. [26] are plotted as individual data points in Fig. 1, where it can be seen that the experimental results do not agree well with the assumed linear relationship between the specific volume and the volumetric fractions near the interception point of the two straight lines. One major discrepancy is that while the theoretical curve has a sharp corner at the interception point of the two straight lines, the experimental curve does not have any sharp corner at the interception point. This deficiency of the 2-parameter models, or rather the linear packing theory, has been discussed before [22–27]. de Larrard [23] attributed such discrepancy to the variation in compaction and developed a compressible packing model, which produces a theoretical curve with no sharp corner. Contrarily, Kwan et al. [26] attributed such discrepancy to a newly identified wedging effect. Meanwhile, alternative approaches using computer simulation [28–33], which in theory are able to account for all structural effects, have been developed. However, because of the high computer power needed, these are more for research than practical applications.

By introducing a wedging effect parameter to account for the wedging effect, which could significantly decrease the packing density, Kwan et al. [26] developed a new packing model wherein the specific volume–volumetric fractions relationship is nonlinear. For the case in which the coarse particles are dominant, the wedging effect occurs when some isolated fine particles are entrapped in the gaps between the coarse particles instead of filling into the voids thereby wedging the coarse particles apart. For the case in which the fine particles are dominant, the wedging effect occurs when some gaps between the coarse particles are too narrow to accommodate one complete layer of fine particles leading to the presence of only isolated fine particles at the gaps wedging the coarse particles apart. With three parameters taking account of the loosening, wall and wedging effects, this model may be classified as the 3-parameter model.

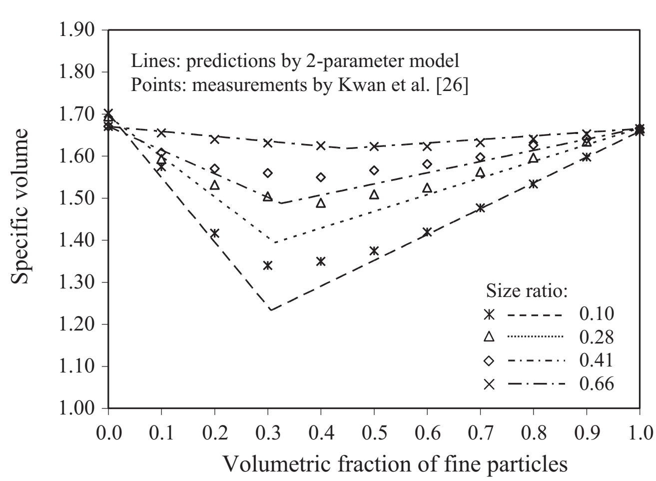


**Fig. 1.** Variation of specific volume with volumetric fractions.

Herein, the above 3-parameter model, originally developed for binary mixes, is modified and extended for application to ternary mixes of spherical particles. This packing model does not rely on the assumption that the specific volume–volumetric fractions relationship is linear and is therefore more advanced than the conventional 2-parameter packing models based on the linear packing theory. Test results from the present study and test results from the literature are used to verify the accuracy and applicability of the extended 3-parameter model for ternary mixes of particles.

## 2. Conventional 2-parameter model for ternary mixes of particles

Consider a ternary mix composed of three size classes of mono-sized particles: size class 1, size class 2 and size class 3, in order of increasing particle size. It is assumed that there is always one dominant size class and there exist only interactions between the dominant size class and each non-dominant size class. Mathematically, the 2-parameter model for ternary mixes consists of three linear equations for packing density prediction, each corresponding to one assumed dominant size class. In the following formulation, the diameters of size class 1, size class 2 and size class 3 are denoted by $d_1$, $d_2$ and $d_3$, respectively (note that $d_1 \le d_2 \le d_3$), the volumetric fractions of size class 1, size class 2 and size class 3 are denoted by $r_1$, $r_2$ and $r_3$, respectively (note that $r_1 + r_2 + r_3 = 1$), and the packing densities of size class 1, size class 2 and size class 3 are denoted by $\phi_1$, $\phi_2$ and $\phi_3$, respectively.

### 2.1. When size class 1 is dominant

Fig. 2 illustrates the packing structure of a ternary mix of particles when size class 1 is dominant. In this case, there would be no particle interaction between size class 2 and size class 3, and the dominant size class 1 would interact with size class 2 and size class 3 independently through the wall effect. The packing density of the ternary mix when size class 1 is dominant (denoted by $\phi_1{}^*$) may be obtained from the following equation:

$$\frac{1}{\phi_{1*}} = \left(\frac{r_1}{\phi_1}+\frac{r_2}{\phi_2}+\frac{r_3}{\phi_3}\right)-(1-b_{12})(1-\phi_2)\cdot\frac{r_2}{\phi_2}-(1-b_{13})(1-\phi_3)\cdot\frac{r_3}{\phi_3} \quad (1)$$

where $b_{12}$ and $b_{13}$ are the wall effect parameters accounting for the wall effects between size class 1 and size class 2, and between size class 1 and size class 3, respectively. The parameters $b_{12}$ and $b_{13}$ are given as

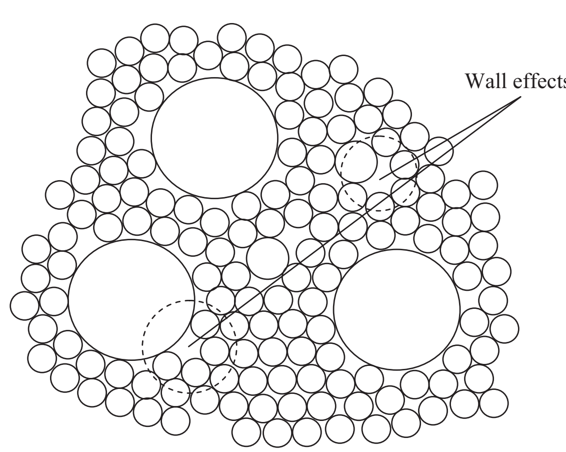


**Fig. 2.** The wall effects when size class 1 is dominant.