Powder Technology 267 (2014) 346–353

Contents lists available at ScienceDirect

Powder Technology

journal homepage: www.elsevier.com/locate/powtec

# Shape of re-entrant particles — characterization regarding particle–fluid interaction

J.M.A.R. de Almeida [a,c], Y. Carvalho [a,c], P.N. Romano [a,c,*], R.P. Peçanha [a,b,c]

[a] Federal University of Rio de Janeiro, School of Chemistry, Brazil
[b] Chemical Engineering Department
[c] Graduate Program in Technology of Chemical and Biochemical Processes, Brazil

ARTICLE INFO

*Article history:*
Received 17 February 2014
Received in revised form 25 May 2014
Accepted 21 July 2014
Available online 7 August 2014

*Keywords:*
Shape factor
Terminal velocity
Sphericity
Re-entrant particle
Correlation
Settling velocity

ABSTRACT

Particle–fluid interaction plays an important role in the design, rating and operational tuning of many industrial equipments (e.g., elutriators, settling chambers, cyclones, hydrocyclones, centrifuges etc.). The core of this interaction is the drag force exerted by the fluid on suspended particles, which, for a given fluid, depends mainly on the fluid-particle relative velocity, as well as on the particle size and shape. Traditionally, particle shape is quantified by the use of shape-factors, the most commonly used being the sphericity. However, sphericity does not discriminate well the effects of particle shape, when the particle surface is markedly re-entrant. In this work the terminal velocities of re-entrant particles, however geometrically simple, were accurately measured while falling under gravity in Newtonian liquids. A total of 77 carefully crafted particles of various materials were tested, at least in triplicate, giving rise to 2798 values of terminal velocities. The strategy was to test particles of the same material, volume and sphericity, but with different re-entrant shapes. Two shape factors were combined with sphericity ($\phi$), to account for the effects of particle shape on particle–fluid interaction. A correlation for the terminal velocity using the three shape factors was developed in terms of drag coefficient ($C_D$) and particle Reynolds number ($Re_p$).



## 1. Introduction

When a particle moves relative to an external surrounding fluid, it experiences a force that for brevity one may call dynamic. For the purpose of analysis, this force is usually split into three orthogonal components as follows: the drag force, parallel to fluid velocity far from the particle, and two forces, both transversal to the particle trajectory, normally called lift forces. In this work we are concerned with particles such that lift forces are negligible compared to drag forces. This situation is, to a great approximation, encountered in many industrial processes including solid–fluid separation, fluidization, pneumatic and hydraulic transport of solids. A practical consequence of this fact is that under an external force like the gravitational, such particles fall in fluids along straight lines. This allows the determination of their terminal velocity ($v_t$) in a simple manner, that is, by measuring vertical distances and time intervals. The associated dimensionless groups known as drag coefficient ($C_D$) and particle Reynolds number ($Re_p$) are then computed with the following classical expressions:

$$C_D = \frac{4}{3}\frac{d_p(\rho_s-\rho)g}{\rho\ v_t^2} \tag{1}$$

$$Re_P = \frac{d_p\ v_t\ \rho}{\mu} \tag{2}$$

where $d_p$ is the particle size expressed as the diameter of the sphere with the same volume as the particle, $\rho_s$ is the particle density, $\rho$ is the fluid density, g is the gravity acceleration and $\mu$ is the fluid viscosity.

The drag force on the particle, and consequently its terminal velocity ($v_t$), also depend on the particle shape, unaccounted for in the above expressions. The effects of this variable on the particle–fluid interaction are quantified by shape factors which can be viewed as a third dimensionless group in addition to $C_D$ and $Re_p$. The most commonly used shape factor is the sphericity ($\phi$), defined as

$$\phi = \frac{\left(\begin{array}{c}\text{surface area of the sphere with}\\ \text{the same volume as the particle}\end{array}\right)}{(\text{surface area of the particle})}. \tag{3}$$

However, the "degree of true sphericity", which is the original name of $\phi$ used by Wadell [1] in representing the shape of sedimentary particles in geological studies, has been criticized on very distinct grounds (Ehrlich and Weinberg [2]; Yin et al. [3]; Chin et al. [4]; Clark et al. [5]; Buffham [6]; Taylor [7]). In this respect, it should be mentioned that Wadell [8] was the first to plot a log-log diagram of $C_D$ versus $Re_p$ using $\phi$ as a parameter.

* Corresponding author.
*E-mail address:* pedronromano@gmail.com (P.N. Romano).

http://dx.doi.org/10.1016/j.powtec.2014.07.050
0032-5910/

Correlations between $C_D$, $Re_p$ and $\phi$ as well as those involving groups like $C_D\ Re_p^2$, which allows the prediction of $v_t$ for a given $d_p$ and $\phi$, and $C_D/Re_p$, which allows the prediction of $d_p$ for a given $v_t$ and $\phi$, abound in the literature (e. g., Pettyjohn and Christiansen [9]; Brown [10]; Haider and Levenspiel [11]; Kunii and Levenspiel [12]; Massarani [13]; Ganser [14]). However, these correlations are fully based on data for non-reentrant particles.

When re-entrant particles are involved, the inadequacy of sphericity to represent the effects of particle shape upon particle–fluid interaction is even greater. In that context, the work of Bouwman et al. [15] illustrates how difficult it is to discriminate the shape of irregular particles by using a single shape factor, such as sphericity. They also have shown that it is a better choice to use a combination of different shape factors when dealing with re-entrant particles. The literature is very scarce regarding such particles. The characterization of their shape was attempted by Staniforth and Rees [16,17] with a dimensionless empirical factor symbolized by the cyrillic letter Ш (pronounced "shah"). The factor is machine generated and, according to the authors, applies best to describe the mean particle shape of bulk powders. More recently Arsenijevic et al. [18] presented a methodology to predict terminal velocities of particles based on the expansion of beds fluidized by liquids. The shape of the particles is implicit on the expansion data, leading to a mean value for the terminal velocity of particles comprising the bed. The method requires the knowledge of the minimum fluidization velocity and porosity and is based on data extrapolation for bed porosities equal to 1.0.

Two important facts related to actual particles (e.g., precipitates, crystals, sands, crushed and ground ores, ashes, soot etc.) present in industrial processes are to be recalled: (a) to some degree, particles are of the re-entrant type; (b) in many equipments (e.g., elutriators, settling chambers, cyclones, hydrocyclones, centrifuges etc.) the concentration of particles in suspension is low and population effects are negligible. In this case the dynamic force acting on the particle inside equipments, depend on its size and shape which, ideally, should account for particle re-entrances. Since for those equipments the component of the particle velocity vector parallel to the prevailing body force is its terminal velocity (except when accelerations are present), accurate predictions of terminal velocities are essential to the design, rating and operational tuning of those equipments. This establishes the research gap that motivated this work.

In order to account for the effects of surface re-entrances on the particle–fluid interaction, the authors suggested to characterize individual particles by means of a parameter called index of convexity ($\eta$) defined as follows:

$$\eta = \frac{\begin{array}{c}\text{smallest volume of material that should be}\\ \text{added to the particle to make it convex}\end{array}}{\text{volume of the particle}}. \tag{4}$$

Notice that $\eta$ is a shape factor such that, for a non re-entrant particle $\eta$ is equal to zero. Clearly $\eta \geq 0$. The index of convexity ($\eta$) was conceived by one of the authors (R. P. Peçanha) and first used by Mendes and Melo [19]. Later it was also used by Almeida, Carvalho and Romano [20]. The procedure to transform a geometrically simple re-entrant particle into a non re-entrant one is depicted in Fig. 1.

Another shape factor used in this work to correlate data on particle–fluid interaction is the index of circumscribed sphere ($\lambda$), originally introduced by Wadell [21] as follows:

$$\lambda = \frac{\text{volume of the particle}}{\begin{array}{c}\text{volume of the sphere that}\\ \text{circumscribes the particle}\end{array}}. \tag{5}$$

Notice that for a spherical particle, the shape factor $\lambda$ is equal to 1. Clearly $0 \leq \lambda \leq 1$. Fig. 2 illustrates a cubic particle and the corresponding circumscribed sphere.

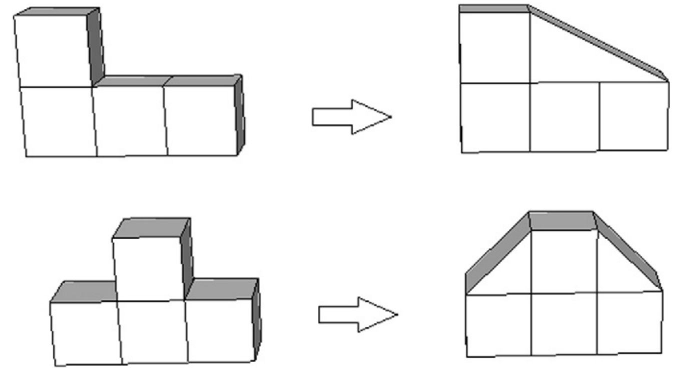

**Fig. 1.** Transformation of re-entrant particles in non re-entrant ones.

This work has a twofold purpose regarding markedly re-entrant particles: (a) to check for the limitations of sphericity in discriminating effects of particle shape upon particle–fluid interaction; (b) to show that sphericity, together with the shape factors $\eta$ and $\lambda$, can correlate particle–fluid interaction, more accurately than sphericity alone.

## 2. Experimental

For a given material, presumably having a constant density, markedly re-entrant particles of relatively simple geometry were craft made so as to have the same volume and surface area, that is, with the same sphericity.

Particles were produced by two methods: (a) differed assembling and (b) bending. These methods, are schematically exemplified, respectively, in Figs. 3 and 4.

In the differed assembling technique, identical sets of fragments were joined together in different ways, so as to give re-entrant particles of different shapes. In Fig. 3, particles A and B were built using two identical fragments leading to particles with the same sphericity but not the same shape. Notice that, unlike Fig. 3 where right prisms were assembled, individual fragments to be joined can have very different shapes.

Assembling of particles required the use of glue. However, the particles were relatively big so that the mass of glue as compared to the mass of the assembled particle was less than 0.001%, that is, a negligible effect regarding particle weight and density. The number of assembled fragments varied from two to four.

Except for an unbalanced area increase (due to stretching in the lower surface) and decrease (due to compression in the upper surface)

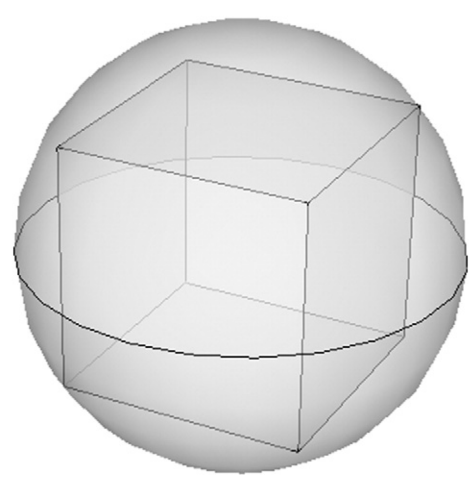

**Fig. 2.** Cubic particle and circumscribed sphere.