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Short communication

Motion of a spherical particle on a rotating parabola using Lagrangian and high accuracy Multi-step Differential Transformation Method

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ABSTRACT

In this letter, the equation of a particle's motion on a rotating parabolic surface is introduced through Lagrange equations and is solved by Multi-step Differential Transformation Method (Ms-DTM). As a main outcome, it is shown that this method gives approximations of a high degree of accuracy and least computational effort for studying particle motion on rotating parabolic surfaces compared to previous analytical methods. Also, position trajectory of the particle, $r(t)$, and its phase planes are depicted in the current study for different constant numbers.

1. Introduction

Many phenomena exist in the environment where a particle's motion can be observed on them, such as centrifugation, centrifugal filters, industrial hopper, etc. Motion surface has different shapes especially for rotating application as it can be circular, parabolic or conical. It's necessary for scientists to analyze the motion of the particles on these surfaces, so an analytical solution is usually the more preferred and convenient method in the field of engineering due to less computational work as well as high accuracy and is widely used for predicting the motion of particles. Some are introduced as follows.

Jalaal et al. [1] solved a spherical particle's motion in Couette flow using homotopy perturbation method (HPM) and got comparable results to numerical ones. The unsteady rolling motion of a spherical particle restricted to a tube was studied analytically by Jalaal and Ganji [2]. They obtained an exact solution of particle velocity and acceleration motion under some practical conditions through applying HPM. Also, Jalaal and Ganji [3] proposed an analytical solution for acceleration motion of a spherical particle rolling down an inclined boundary with drag coefficient which is correlated linearly to *Re* in a specific range using HPM. They studied various inclination angles and observed that settling velocity, acceleration duration and displacement are proportional to amount of inclination angle while for a constant inclination angle; settling velocity and acceleration duration are decreased by increasing the fluid viscosity [3]. Torabi and Yaghoobi [4] investigated the better performance of a combination of He's polynomials and the diagonal Padé approximants rather than using the HPM which has been shown for calculating approximate solution of the acceleration motion of a single spherical particle moving in a continuous fluid phase. In the last study, they obtained the acceleration trajectory of a non-spherical particle moving in a continuous fluid phase using VIM–Padé approximants method with acceptable accuracy compared to numerical results [5]. Recently, Ghasemi et al. [6] discussed about the convergence and accuracy of VIM and ADM for solving the motion of a spherical particle in Couette fluid flow. Marinca et al. [7] solved the equation of motion a particle on a rotating parabola using OHAM and recently Mirzabeigy et al. [8] presented other analytical methods such as energy balance for this problem but their methods need an improvement for increasing the convergence.

Analytical methods are powerful techniques for solving the nonlinear differential equations in physical problems. For instance, Hatami and Ganji [9–13] applied different analytical methods in heat transfer problems. Hatami et al. [14] used HAM [15–17] for solving the natural convection of nanofluid over a horizontal plate. Also, Hatami et al. [18,19] used weighted residual function methods for solving fluids flow problems. Recently, Hatami and Domairry [20,21] applied DTM with Pade approximation for predicting the treatment of soluble particles in Newtonian media. Multi-step Differential Transformation Method (Ms-DTM) which is a modified form of classical DTM introduced by Zhou [22], can be a suitable method for this kind of engineering and

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physical problem. This method is newly used in some engineering problems [23–25].

As revealed in the above section, the need for a high efficient technique to predict the particle motion on a rotating parabola is completely obvious. The main objective of the present study is to introduce the Ms-DTM as a high accuracy and efficient method for motion of a spherical particle on a rotating parabolic surface.

2. Statement of problem

Consider a particle which slides along a surface that has the shape of a parabola $z = cr^2$ (see Fig. 1). The following assumptions are considered for particles motion modeling:

- Particle is at equilibrium.
- The particle rotates in a circle of radius R.
- The surface is rotating about its vertical symmetry axis with angular velocity ω.

By choosing the cylindrical coordinates r, θ, and z as generalized coordinates, the kinetic and potential energies are,

$$T = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{z}^2\right). \qquad U = mgz \tag{1}$$

We have in this case some equations of constraints that we must take into account, namely

$$z = cr^2 \qquad \dot{z} = 2c\dot{r}r \tag{2}$$

and

$$\theta = \omega t \qquad \dot{\theta} = \omega \tag{3}$$

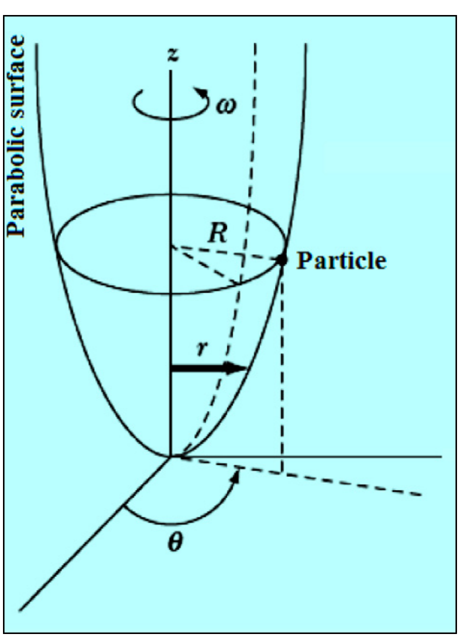

Fig. 1. Schematic view of a spherical particle on a rotating parabolic surface.

Inserting Eqs. (3) and (2) in Eq. (1), we can calculate the Lagrangian for the problem

$$L = T - U = \frac{1}{2}m\left(\dot{r}^2 + 4c^2r^2\dot{r}^2 + r^2\omega^2\right) - mgcr^2. \tag{4}$$

It is important to note that the inclusion of the equations of constraints in the Lagrangian has reduced the number of degrees of freedom to only one, i.e., r. We now calculate the equation of motion using Lagrange's equation

$$\frac{\partial L}{\partial r} = m\left(4c^2r\dot{r}^2 + r\omega^2 - 2gcr\right) \qquad \frac{d}{dt}\frac{\partial L}{\partial \dot{r}} = m\left(\ddot{r} + 4c^2r^2\ddot{r} + 8c^2r\dot{r}^2\right) \tag{5}$$

and

$$\ddot{r}\left(1 + 4c^2r^2\right) + \dot{r}^2\left(4c^2r\right) + r\left(2gc - \omega^2\right) = 0. \tag{6}$$

Nayfeh and Mook [26] considered that $2gc - \omega^2 = \varepsilon^2$ and so

$$\ddot{r} + 4c^2\ddot{r}r^2 + 4c^2r\dot{r}^2 + \varepsilon^2 r = 0. \tag{7}$$

It's considered that initial particle position is in radius A and its initial velocity is zero. So, its initial conditions are:

$$r(0) = A, \quad \dot{r}(0) = 0. \tag{8}$$

3. Multi-step Differential Transformation Method (Ms-DTM)

Multi-step Differential Transformation Method due to some advantages is applied in physical applications such as Refs. [23–25]. For example, Ms-DTM due to small time steps has a powerful accuracy especially for initial value problems. Also, because it's based on DTM does not need to small parameter, auxiliary function and parameter, discretization, etc. versus other analytical methods. For perception of Ms-DTM basic idea, consider a general equation of n-th order ordinary differential equation [23],

$$f\left(t, y, y', \ldots, y^{(n)}\right) = 0 \tag{9}$$

subject to the initial conditions

$$y^{(k)}(0) = d_k, \quad k = 0, \ldots, n-1. \tag{10}$$

To illustrate the differential transformation method (DTM) for solving differential equations, the basic definitions of differential transformation are introduced as follows. Let $y(t)$ be analytic in a domain D and let $t = t_0$ represent any point in D. The function

Table 1
Some fundamental operations of the differential transform method.

Origin function	Transformed function
$x(t) = \alpha f(x) \pm \beta g(t)$	$X(k) = \alpha F(k) \pm \beta G(k)$
$x(t) = \frac{d^m f(t)}{dt^m}$	$X(k) = \frac{(k+m)!F(k+m)}{k!}$
$x(t) = f(t)g(t)$	$X(k) = \sum_{l=0}^{k} F(l)G(k-l)$
$x(t) = t^m$	$X(k) = \delta(k-m) = \begin{cases} 1, & \text{if } k = m, \\ 0, & \text{if } k \neq m. \end{cases}$
$x(t) = \exp(t)$	$X(k) = \frac{1}{k!}$
$x(t) = \sin(\omega t + \alpha)$	$X(k) = \frac{\omega^k}{k!}\sin\left(\frac{k\pi}{2} + \alpha\right)$
$x(t) = \cos(\omega t + \alpha)$	$X(k) = \frac{\omega^k}{k!}\cos\left(\frac{k\pi}{2} + \alpha\right)$