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Investigation of anisotropic dispersion in a discontinuous solid mixer

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ABSTRACT

Solid mixing processes are usually described with the help of the dispersion model. Dispersion is based on stochastic particle movements which are responsible for the concentration balance between the different components. For simplification, homogeneity adjustment of the mixture is mostly described by the axial mixing direction. In this research, the one-dimensional dispersion model is extended to three dimensions. A modified sampling strategy shows how the dispersion model can be applied to the mixing process in axial, radial, and tangential directions. Small differences between axial and radial mixing efficiencies are observed for the 45°-configuration of the mixing tool, whereas a higher mixing efficiency is detected in tangential direction. The 90°-configuration behaves differently. Anisotropic dispersion coefficients are determined for the different mixing tool configurations. These indicate that particle mobility in axial direction is lowest.

1. Introduction

To describe various solid mixing processes, different model equations are used. The best-known and most widely used model for solid mixing systems is the stochastic dispersion–convection model, which was derived from Fokker [1] and Planck [2].

The dispersion–convection model is frequently used in a one-dimensional mode to describe the mixing processes in continuous mixers [3–6]. It is shown that the mixing mechanisms in a continuous dynamic mixing apparatus can be described well by the Fokker–Planck Equations with the simplifying assumption of ideal mixing being very quickly for the radial and tangential directions. This simplification is justified if the path lengths differ significantly in the three spatial directions and if, as a result, mixing is influenced by the longest path length. For instance, when considering a mixing apparatus in which the length by far exceeds the diameter [3–5], the mixing process is dominated most significantly by the axial direction. In this case, the changes in the system can be described adequately by one local coordinate.

The one-dimensional dispersion model is also applied for the description of discontinuous solid mixing processes [7–14]. It is found that the axial dispersion model can be used for the characterization of particle mobilities in different mixing processes. Sommer [7] presents a detailed explanation, derivation, and application of the axial dispersion model in a batch mixing process. In rotating drum mixers, the mixing mechanism is considered to result from axial particle movements due to dispersion [8–11]. Different correlations of particle mobilities and particulate materials, operating conditions, and mixing devices are described. The model is also used to describe the blending of irregularly shaped particles [12] and the mixing of segregating particles [13], and to determine the mixing efficiencies in a twin-shaft paddle mixer [14]. In the reported researches [7–14], it is assumed that mixing along the mixer axis decisively influences the duration of mixing. If the path length in axial direction is the largest by far, it is reasonable to assume that reaching homogeneity is affected by axial mixing. However, when a batch mixing process with similar path lengths in axial and radial directions is considered, the one-dimensional view needs to be discussed. So far, no qualitative data have been provided as to how the mixing differs in the three spatial directions in a discontinuous solid mixer with a rotating mixing tool. If mixing differs considerably as a function of the directions, the direction with the smallest particle mobility is sufficient to describe the mixing process. In the opposite case, the multi-dimensional model needs to be considered.

2. Dispersion model

The dispersion model, also called diffusion–convection model, is based on Fick's second law with an additional convective term. In three-dimensional notation, the dispersion model is described by Eq. (1):

$$\frac{\partial c(\xi,t)}{\partial t} = \underbrace{\Delta[c(\xi,t)\cdot D(\xi,t)]}_{\text{diffusion}} - \underbrace{\nabla[c(\xi,t)\cdot U(\xi,t)]}_{\text{convection}}, \tag{1}$$

where:

c = concentration of one mixing component,
ξ = local coordinate (substitute for z, r or θ),
t = time,
D = dispersion or diffusion coefficient [length^2/time],
U = transport coefficient [length/time].

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The model describes the changes in concentration c as a function of the location and the time t due to a convective transport of particle collectives and random particle motions. The mixing mechanism caused by random particle motions is referred to as dispersion, whereas convection leads to large-scale distribution by directed motions of the material. Convective particle transport is characterized by the transport coefficient U with the dimension of speed. To distinguish Fickian diffusion in the gas or liquid phase from the stochastic motion of particles in solids, the dispersion coefficient D is used. Both the diffusion coefficient of fluids and the dispersion coefficient of solids are mathematically the same and characterize the mobility of particulate material. While the mobility of molecules in fluids depends on the temperature, the mobility of particles in bulk materials depends on their physical properties (shape, surface, size) as well as on the introduced kinetic energy. Without external energy, the particles in bulk materials fix each other in their positions. This leads to the difficulty of separating the particle motions into dispersive and convective movement processes. According to Daumann et al. [14], the dispersion coefficient is therefore an integral parameter in batchwise operating mixers, which describes the homogenization in concentration by random particle movements resulting from the rotating mixing tools.

If the mixed particles differ in their movement behavior because of differences in particle size and shape or density, the homogenization process can be influenced by segregation effects. Due to higher particle mobilities, for instance, smaller particles are transported faster to the bottom of the mixer than bigger ones. Because of such effects, the mixing process can be influenced by segregation. The transport coefficient is used as a measure of segregation effects caused by convective particle transportation [7,15]. As long as the physical properties of the two particle components are the same, the dispersion model can be used without the convective term to characterize a mixing process [12,14]. This indicates that the concentration balance during the mixing process is characterized by purely dispersive mixing. For time-independent coefficients, neglecting the convective term and adaptation to a cylindrical mixing vessel, the dispersion–convection Eq. (1) transforms to [16]:

$$\frac{\partial c(z,r,\theta,t)}{\partial t} = \underbrace{\frac{\partial^2 c(z,t)}{\partial z^2} D_Z}_{\text{axial}} + \underbrace{\left(\frac{1}{r}\frac{\partial c(r,t)}{\partial r} + \frac{\partial^2 c(r,t)}{\partial r^2}\right) D_r}_{\text{radial}} + \underbrace{\left(\frac{1}{r^2}\frac{\partial^2 c(\theta,t)}{\partial \theta^2}\right) D_\theta}_{\text{tangential}}, \tag{2}$$

where:

$\theta =$ tangential coordinate, angle [°],
$z =$ axial coordinate [mm],
$r =$ radial coordinate [mm],
$D_\xi = {}_{z,r,\theta} =$ axial, radial, and tangential dispersion coefficient [mm^2/s].

Eq. (2) describes the dispersion in all three directions at the same time. For comparison and determination of the three different dispersion coefficients each direction is considered individually. Fig. 1 illustrates the three different initial conditions used in the experiments to compare axial, radial, and tangential mixing. The red tracer particles as well as the transparent filler particles are added to the mixing vessel with self-made devices in such a way that the particles can be seen perfectly separated.

2.1. Axial dispersion

If only the axial particle movements are considered, the spatial coordinates r and θ disappear and Eq. (2) simplifies to axial dispersion Eq. (3):

$$\frac{\partial c(z,t)}{\partial t} = \frac{\partial^2 c(z,t)}{\partial z^2} D_Z. \tag{3}$$

For Eq. (3), a variety of solutions results for different initial conditions. An overview of initial conditions is given by Sherritt et al. [10]. To investigate axial dispersion, a thin layer of red tracer particles is added perpendicularly to the mixer axis z at the top of the bulk material as shown in Fig. 1a. At the beginning of the mixing process, all tracer particles ($c = 1$) are located on the surface of the bulk material, while the absence of tracer particles in the bulk material is defined by $c = 0$. The step function in Eq. (4) precisely characterizes the initial conditions of the experimental procedure:

$$\begin{aligned} c(z,0) &= 1, \quad 0 \le z \le Z_i, \\ c(z,0) &= 0, \quad Z_i < z \le Z_0, \end{aligned} \tag{4}$$

where Z_0 corresponds to the height of the bulk material in the mixing vessel and Z_i is the border radius between the differently colored particle fractions. The average tracer concentration $\bar{c}$ and Z_i correlate with the as $\bar{c} = Z_i/Z_0$. No dispersion at the surface of the bulk material and the bottom of the mixing vessel is described by disappearing concentration gradients at all points of the surface and the bottom:

$$\left.\frac{\partial c(z,t)}{\partial z}\right|_{z=0} = 0, \quad \left.\frac{\partial c(z,t)}{\partial z}\right|_{z=Z_0} = 0. \tag{5}$$

The boundary conditions ensure that the concentration tends to the average concentration with increasing mixing time. Under the initial and boundary conditions mentioned, the solution of axial dispersion Eq. (3) is given by [11]:

$$c(z,t) = \bar{c} + \frac{2}{\pi}\sum_{n=1}^{\infty} e^{-n^2\pi^2\frac{D_Z t}{Z_0^2}} \frac{\sin(n\pi\bar{c})}{n} \cos\left(n\pi\frac{z}{Z_0}\right), \tag{6}$$

where n is an integer and D_Z the axial dispersion coefficient. If the average tracer concentration is given and the height of the bulk material is constant, the concentration depends on the position, mixing time, and axial dispersion coefficient. The concentrations versus axial positions and mixing time for an arbitrary dispersion coefficient are plotted in Fig. 2a in order to illustrate Eq. (6). In the latter, the selected values of the variables $\bar{c}, Z_0, Z_i$ correspond to the dimensions of the experimental procedure. At the beginning, all tracer particles are located on the surface of the bulk material. With the passage of time, the tracer particles distribute in the bulk over the entire height of the mixture until a uniform concentration $c(z,t) = \bar{c}$ is reached at all positions. The concentration tends rapidly or slowly to the average tracer concentration as the dispersion coefficient is selected high or low, respectively. A high dispersion coefficient means quickly disappearing gradients, which in turn describe rapid attainment of homogeneity.

2.2. Radial dispersion

Considering only the radial direction by neglecting the coordinates z and θ, the dispersion model simplifies to Eq. (7):

$$\frac{\partial c(r,t)}{\partial t} = \left(\frac{1}{r}\frac{\partial c(r,t)}{\partial r} + \frac{\partial^2 c(r,t)}{\partial r^2}\right) D_R. \tag{7}$$

Eq. (7) has already been used for modeling the mixing of particulate solids accompanied by segregation in a horizontally rotating drum mixer [17]. Therein, particle concentration is described as a function of the time and the position for a radially mixed two-component system. Under certain initial and boundary conditions, Eq. (7) can be solved

Table 1
Zeros of the Bessel function $J_1(\lambda_n R_0) = 0$.

$\lambda_1 R_0$	3.832
$\lambda_2 R_0$	7.016
$\lambda_3 R_0$	10.173
$\lambda_4 R_0$	13.324
$\lambda_5 R_0$	16.471