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# Specific energy consumption and optimum operating condition for coarse-particle slurries

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This paper deals with settling solid–liquid (slurry) flows of coarse particles with particle sizes of 150 μm or larger. The specific energy consumption (SEC), which is a measure of transport efficiency, was used to find the optimum operating condition. The effect of different parameters on SEC is determined by using the near-wall lift model. The analysis shows that the minimum SEC occurs at a solids concentration of approximately 30% by volume. The accuracy of the near-wall lift model is investigated by comparing the model predictions with the results of a number of experimental works. The comparisons show good agreement between model prediction and experimental data in the literature.

## 1. Introduction

Solid–liquid (slurry) flows are widely used in many industrial processes and determining the optimum flow condition is an important factor in pipeline design and operation. Slurry flows are generally divided into two groups based on fluid and particle characteristics: non-settling or homogenous, and settling or heterogeneous slurry flows [1].

One of the important factors in determining the operating condition for a slurry flow is energy consumption. The specific energy consumption (SEC) is a measure of energy required to transport a unit mass of solids over a unit pipeline length and can be written as:

$$SEC = \frac{i_m}{S_s C_{vd}} \tag{1}$$

where $i_m$, $S_s$ and $C_{vd}$ are hydraulic gradient, relative solids density (solids to fluid density ratio) and delivered concentration. Lower SEC values represent more energy-effective operation and consequently, more efficient transport [2]. The hydraulic gradient is the frictional head loss in terms of height of carrier fluid per unit length of pipe and defined as:

$$i_m = \frac{1}{\rho_f g}\left(-\frac{\Delta p}{\Delta x}\right) \tag{2}$$

where $-\frac{\Delta p}{\Delta x}$ and $\rho_f$ are the pressure drop per unit length of pipe and fluid density, respectively. The pressure gradient could also be expressed as the column height of slurry per unit length, $j_m$, which can be written as:

$$j_m = \frac{1}{\rho_m g}\left(-\frac{\Delta p}{\Delta x}\right) \tag{3}$$

where $\rho_m$ is the density of the mixture and is a linear function of solids volume concentration ($Cs$).

$$\rho_m = \rho_s C_s + \rho_f (1 - C_s) \tag{4}$$

An accurate prediction of SEC for a slurry flow relies on having a reliable model to calculate hydraulic gradient or pressure gradient.

Heterogeneous slurry flows typically occur when coarse or settling particles are being transported. With these slurries, there is a minimum operating velocity which is required to avoid particle accumulation and a stationary deposit in the pipe. They are also characterized by asymmetric concentration and velocity profiles in horizontal flows [3].

Two major frictional loss mechanisms in coarse particle slurry flows are: *(1)* particle dispersive stresses and *(2)* Coulombic or contact–load friction. The particle dispersive stresses are caused by shear-related particle interactions. Shearing the closely-spaced particles generates a normal stress which can be correlated to shear rate, solids concentration and particle size [4]. These forces tend to drive particles toward the pipe wall. They are strongly dependent on particle concentration and are dominant at high solids concentrations. The Coulombic stress is due to particles which are not suspended by fluid turbulence. The immersed weight of non-suspended particles is supported through particle

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contact with the pipe wall. This force is strongly dependent on particle size and independent of pipeline velocity [5].

Durand & Condolios [6] developed a relation for calculation of hydraulic gradient for heterogeneous slurries. They represented the hydraulic gradient by using a dimensionless parameter $\Phi$, defined as:

$$\Phi = \frac{i_m\text{-}i_w}{C_{vd} i_w} \tag{5}$$

where $i_w$ is the hydraulic gradient for the single phase flow of the fluid under identical conditions (i.e. velocity, pipe diameter). They related $\Phi$ to another dimensionless variable, $\Psi$ which is defined as:

$$\Psi = \frac{{V_m}^2}{gD(S_s - 1)}\sqrt{C_D} \tag{6}$$

where $V_m$ and $C_D$ are the mixture velocity and particle drag coefficient, respectively. Newitt et al. [7] suggested that for heterogeneous slurries the hydraulic gradient can be obtained using the stratification ratio provided that $V_m < 17v_t$ where $v_t$ is the particle terminal settling velocity. The stratification ratio, $R$, is expressed as:

$$R = \frac{i_m - i_w}{S_m - 1} \tag{7}$$

where $S_m$ is the relative mixture density ($\rho_m/\rho_f$). They proposed that for these types of slurries the stratification ratio is constant and is equal to 0.8. In this case the main problem is the fact that the hydraulic gradient is not a function of particle size. Gibert [8] developed the following expression for determining hydraulic gradient in heterogeneous slurries. He proposed that:

$$i_m = i_w(1 + C_t\varphi) \tag{8}$$

where $C_t$ is the solids mass concentration and $\varphi$ is:

$$\varphi = 180\left\{\frac{V^2}{gD}\sqrt{C_x}\right\}^{-\frac{3}{2}} \tag{9}$$

The parameter $C_x$ is a fictitious drag coefficient and expressed as:

$$\sqrt{C_x} = \sqrt{\frac{gd_p}{v_t}} \tag{10}$$

where $d_p$ is the particle diameter. He analyzed the energy consumption in a slurry flow and found that the energy consumption is inversely proportional to the solids concentration and has its lowest value at a point which has the minimum value in the hydraulic gradient versus pipeline velocity curve.

Although these models were mostly correlations based on many experimental results, and some of the functionalities were not correct, they provide valuable insight in modeling and design of slurry pipelines.

Extensive analysis of heterogeneous slurries at Queen's University [9,10] and Saskatchewan Research Council (SRC) Pipe Flow Technology Centre [1,11] resulted in subsequent development of slurry flow models such as the SRC Pipe Flow model. These models are mainly based on the contribution of particle dispersion by fluid turbulence and Coulombic friction.

The concentration profiles predicted by these models show a monotonic increase in concentration toward the bottom of the pipe in horizontal flows. The experimental results of horizontal slurry flows of coarser particles obtained by various researchers [12–14] produced solid concentration profiles with a maximum concentration in the lower section of the conduit. These results contradicted the prediction of the models and showed that another significant mechanism(s) is/are responsible for driving particles away from the wall.

Wilson et al. [15,16] showed that at high velocities, particles experience a lift force which results in particle migration away from the wall. They showed that this force is effective only near the wall. It is strongly dependent on the shape of fluid velocity profile and the ratio of the particle diameter to viscous sublayer thickness. As well, the effect of this near-wall lift force is greatest for coarse particles at high velocities.

Wilson et al. [17] developed a new model for heterogeneous slurry flows which includes the near-wall lift force. In their model they mainly focused on the near-wall region and neglected the core of the flow since the main resistance to flow occurs near the pipe wall. They found a good agreement between the model prediction and a large body of experimental data.

Hashemi et al. [18] studied the SEC and desirable operating conditions for fine-particle slurries. They analyzed aqueous mixtures of inert fine particles, typically 100 μm or smaller, but not so small as to cause significant non-Newtonian viscous effects. They used the equivalent-fluid model to predict hydraulic gradient and SEC and showed that this model provides reasonable predictions in slurries of this type.

In the present paper, the Wilson et al. [17] model is used to determine the SEC and optimum operating condition for heterogeneous slurry flows. The slurries of interest are heterogeneous slurries of coarse solids with particle diameters greater than 150 μm. Effects of different parameters such as particle size, pipe diameter, solids concentration and mixture velocity on SEC and optimum operating condition are all investigated.

## 2. Analysis

Wilson et al. [17] considered two models for slurry pipeline flows. These two models are the equivalent-fluid model for homogenous slurry flows and the near-wall lift model for heterogeneous slurry flows. The equivalent-fluid model is typically use for fine particle slurries and assumes that the mixture behaves as a single phase fluid with a modified density. The assumption is that particles are not so small to cause non-Newtonian behavior and that they are distributed uniformly in the flow. This model can be used for solids-liquid mixtures with particle diameters of 100 μm or smaller.

The heterogeneous model which is the focus of the present paper is applicable to a solids-liquid mixture with larger particles where the turbulence dispersion force does not fully maintain the particle suspension and results in asymmetric concentration profiles. Wilson et al. [17]

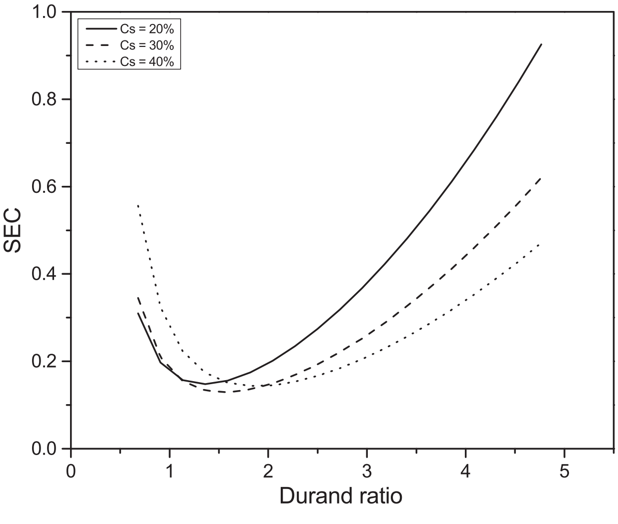

**Fig. 1.** The effect of solids concentration on SEC ($d_p = 150$ μm, $D = 0.15$ m).