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Development of empirical models with high accuracy for estimation of drag coefficient of flow around a smooth sphere: An evolutionary approach

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ABSTRACT

An accurate correlation for the smooth sphere drag coefficient with wide range of applicability is a useful tool in the field of particle technology. The present study focuses on the development of high accurate drag coefficient correlations from low to very high Reynolds numbers (up to 10^6) using a multi-gene Genetic Programming (GP) procedure. A clear superiority of GP over other methods is that GP is able to determine the structure and parameters of the model, simultaneously, while the structure of the model is imposed by the user in traditional regression analysis, and only the parameters of the model are assigned. In other words, in addition to the parameters of the model, the structure of it can be optimized using GP approach. Among two new and high accurate models of the present study, one of them is acceptable for the region before drag dip, and the other is applicable for the whole range of Reynolds numbers up to 10^6 including the transient region from laminar to turbulent. The performances of the developed models are examined and compared with other reported models. The results indicate that these models respectively give 16.2% and 69.4% better results than the best existing correlations in terms of the sum of squared of logarithmic deviations (SSLD). On the other hand, the proposed models are validated with experimental data. The validation results show that all of the estimated drag coefficients are within the bounds of $\pm 7\%$ of experimental values.

1. Introduction

The motion of particles in fluids is a key subject in many problems in the fields of chemical and metallurgical engineering as well as mechanical and environmental engineering. The solution of these problems generally involves determining the local behavior of flow and the interaction between solid and liquid phases through the knowledge of hydrodynamic forces such as drag. The drag force is the combination of the normal (i.e. pressure) and tangential (i.e. wall shear stress) forces on the body in the flow direction. However, the distributions of the pressure and wall shear stress are often very difficult to achieve, so the magnitude of the drag force can be determined only through the knowledge of drag coefficient. Analytical determination of the drag coefficient such as Stokes' law is only valid for Reynolds number, Re, less than 0.1 (Flemmer and Banks [1], Kreith [2]), although the drag coefficient can be ascertained using empirical and semi-empirical correlations based on experimental data when inertial effects are significant (i.e. higher Reynolds numbers).

The drag coefficient of a smooth sphere in incompressible flow is a function of Re based on both theoretical investigations and numerous experimental data (Kreith [2]). The main classes of the dependence of drag coefficient on Reynolds number are (1) very low Reynolds number flow (i.e. creeping flow), (2) moderate Reynolds number flow (i.e. laminar boundary layer), and (3) very large Reynolds number flow (i.e. turbulent boundary layer) (Munson et al. [3]). In the first class ($Re < 1$), the flows reflect entirely the viscous effect of flow with no separation results. By increasing Reynolds numbers (i.e. increasing the particle size or flow velocity for a given Kinematic viscosity), the separation region can be observed at $Re \approx 10$, and the region increases until $Re \approx 1000$, where most of the drag is due to pressure drag rather than frictional drag. Parenthetically, it should be noted that the value of the drag coefficient decreases, as wake area becomes larger. At a sufficiently high Reynolds number ($10^3 < Re < 10^5$), the drag coefficient is relatively constant (Munson et al. [3]). When transition from laminar to turbulent flow occurs, a dramatic dip (up to almost 80%) in the drag coefficient appears at critical value $Re \approx 2 \times 10^5$ since the turbulent boundary layer travels further along the surface into the adverse pressure gradient on the rear portion of the sphere before the separation, so the wake is smaller, causing less pressure drag. After

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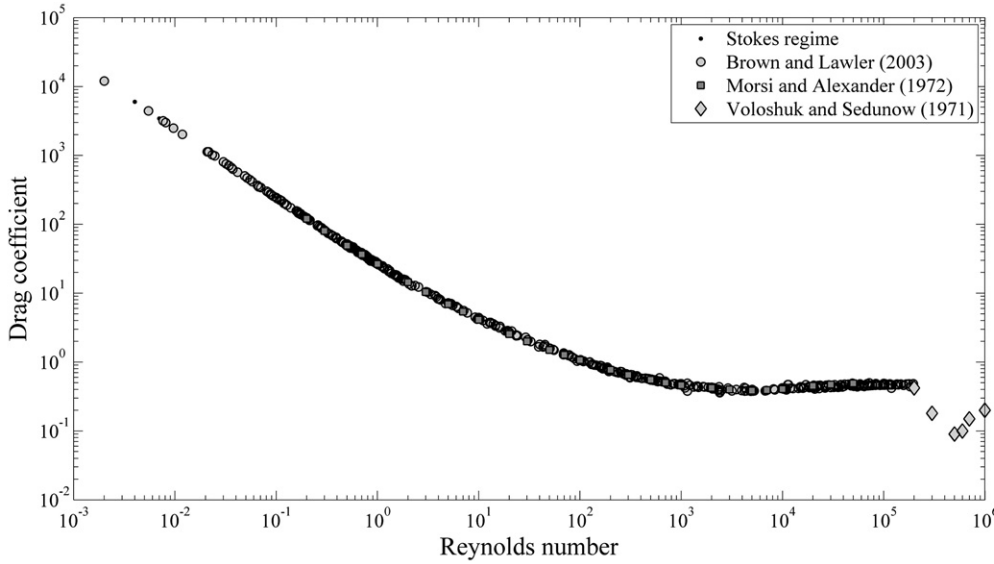

Fig. 1. Illustration of the variations of the drag coefficient with Reynolds numbers using reliable data points of Stokes regime and available experiments in the literature [5,8,11].

Table 1
Summary of some empirical relationships for $Re < 2 \times 10^5$.

Investigator	Model and Reynolds number range	Equation no.
Rouse [14]	$\hat{C}_D = \frac{24}{Re} + \frac{3}{Re^{0.5}} + 0.34 \text{ for } Re<2 \times 10^5$	(1)
Engelund and Hansen [13]	$\hat{C}_D = \frac{24}{Re} + 1.5 \text{ for } Re<2 \times 10^5$	(2)
Clift and Gauvin [17][a]	$\hat{C}_D = \frac{24}{Re}\left(1 + 0.152Re^{0.677}\right) + \frac{0.417}{1+5070Re^{-0.94}} \text{ for } Re<2 \times 10^5$	(3)
Morsi and Alexander [11]	$\hat{C}_D = \begin{cases} \frac{24}{Re} & \text{for } Re<0.1, \\ \frac{22.7300}{Re} + \frac{0.0903}{Re^2} + 3.6900 & \text{for } 0.1<Re<1, \\ \frac{29.1667}{Re} - \frac{3.8889}{Re^2} + 1.2220 & \text{for } 1<Re<10, \\ \frac{46.5000}{Re} - \frac{116.6700}{Re^2} + 0.6167 & \text{for } 10<Re<100, \\ \frac{98.3300}{Re} - \frac{2778}{Re^2} + 0.3644 & \text{for } 100<Re<1000, \\ \frac{148.6200}{Re} - \frac{4.75\times10^4}{Re^2} + 0.3570 & \text{for } 1000<Re<5000, \\ -\frac{490.5460}{Re} + \frac{57.87\times10^4}{Re^2} + 0.4600 & \text{for } 5000<Re<10,000, \\ -\frac{1662.5000}{Re} + \frac{5.4167\times10^6}{Re^2} + 0.5191 & \text{for } 10,000<Re<50,000 \end{cases}$	(4)
Graf [36]	$\hat{C}_D = \frac{24}{Re} + \frac{7.3}{1+Re^{0.5}} + 0.25 \text{ for } Re<2 \times 10^5$	(5)
Flemmer and Banks [1][a]	$\hat{C}_D = \frac{24}{Re}10^E \text{ where } E = 0.383Re^{0.356} - 0.207Re^{0.396} - \frac{0.143}{1+(\log Re)^2} \text{ for } Re<2 \times 10^5$	(6)
Khan and Richardson [20][a]	$\hat{C}_D = \left(2.49Re^{-0.328} + 0.34Re^{0.067}\right)^{3.18} \text{ for } Re<2 \times 10^5$	(7)
Swamee and Ojha [21]	$\hat{C}_D = 0.5\left\{16\left[\left(\frac{24}{Re}\right)^{1.6} + \left(\frac{130}{Re}\right)^{0.72}\right]^{2.5} + \left[\left(\frac{40,000}{Re}\right)^2 + 1\right]^{-0.25}\right\}^{0.25} \text{ for } Re<1.5 \times 10^5$	(8)
Yen [37]	$\hat{C}_D = \frac{24}{Re}\left(1 + 0.15\sqrt{Re} + 0.017Re\right) - \frac{0.208}{1+10^4Re^{-0.5}} \text{ for } Re<2 \times 10^5$	(9)
Haider and Levenspiel [19][a]	$\hat{C}_D = \frac{24}{Re}\left(1 + 0.150Re^{0.681}\right) + \frac{0.407}{1+8710Re^{-1}} \text{ for } Re<2 \times 10^5$	(10)
Cheng [6]	$\hat{C}_D = \frac{24}{Re}(1 + 0.27Re)^{0.43} + 0.47\left[1 - \exp\left(-0.04Re^{0.38}\right)\right] \text{ for } Re<2 \times 10^5$	(11)
Terfous et al. [22]	$\hat{C}_D = 2.689 + \frac{21.683}{Re} + \frac{0.131}{Re^2} - \frac{10.616}{Re^{0.1}} + \frac{12.216}{Re^{0.2}} \text{ for } 0.1<Re<5 \times 10^4$	(12)
Mikhailov and Freire [7]	$\hat{C}_D = \frac{3808\left[(1,617,933/2030)+(178,861/1063)Re+(1219/1084)Re^2\right]}{681Re\left[(77,531/422)+(13,529/976)Re-(1/71,154)Re^2\right]} \text{ for } Re<118,300$	(13)

[a] These models were improved by Brown and Lawler [5].