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# Finite element modelling of wall pressures in a cylindrical silo with conical hopper using an Arbitrary Lagrangian–Eulerian formulation

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ABSTRACT

Silos, especially the ones consisting of cylindrical section with conical hopper, are commonly used for bulk material handling in many industries. Whilst the pressure acting on silo walls during filling is reasonably well understood, a reliable prediction of pressure during discharge remains an important open problem for silo design. This paper describes a finite element analysis of the granular flow in a model silo consisting of a cylindrical section with a conical hopper. The computations were performed using an Arbitrary Lagrangian–Eulerian formulation with an explicit time integration approach to permit large deformations and overcome mesh distortion problems. The finite element results of silo pressure were temporally averaged and compared with the experimental observations in a model silo, which shows a satisfactory agreement in the wall pressure distribution. Two critical modelling issues have been addressed in some detail: one is the numerical treatment of the abrupt transition between the cylindrical section and the conical hopper, and the other is the interaction between the granular solid and the silo walls that is modelled using a dynamic friction model in this study. The simulation results show significant pressure fluctuations during silo discharge, which are comparable to the fluctuating pressure patterns reported in previous experiments. Two dominant frequencies are identified from the dynamic pressure, and a scrutinization of the simulation results suggests that the causes may be attributable to the propagation of the longitudinal waves within the stored granular solid and the intermittent macro-slip of the granular solid against the silo wall. These dynamic events could be a source of silo quaking and honking phenomena.



## 1. Introduction

Silos consisting of cylindrical bin with conical hopper are a common form of containment structure for the storage and handling of industrial bulk solids. The pressures exerted on the silo walls during filling and discharge need to be evaluated for the design and safe operation of these silos; however, there is still a lack of clear description of the wall pressure, especially during discharge. Simple analytical models published in the past seem to be insufficient to describe the complex behaviour of the granular material stored in silos. More sophisticated numerical simulations that take into account aspects such as the silo geometry, stress–strain relation of the granular material, interaction between the granular solid and the silo walls, as well as the emptying process, could provide a much greater capability for investigating the associated complex phenomena.

The last two decades have seen many attempts at developing finite element (FE) models to represent the behaviour of granular materials during silo discharge (e.g. [1–4]). However, apart from the difficulty with mesh distortion caused by large deformations, it has been evident that the continuum FE method has other limitations in simulating granular flow in such silos. For instance, it is difficult to model flowing granular solid against an abrupt change of direction at the transition from the vertical section to the converging section [5,6]. The numerical difficulty with the singularity at the transition has not been addressed fully in the literature. Existing continuum approaches often circumvent the problem by using a conical hopper and flat-bottomed silo to separately examine the silo flow [3,7].

Another aspect that is of particular importance to the numerical computation of silo wall pressure is the interaction between the silo structure and the stored material. According to the Janssen's theory [8], the wall pressure approaches an asymptotic value towards the bottom of a tall silo and this asymptotic value depends strongly on the coefficient of wall friction. Traditionally, the wall frictional behaviour can be described by the Coulomb friction model, relating frictional traction and normal pressures on the walls through a constant friction coefficient. Such an approach is sufficient to describe the frictional behaviour for silo filling and storage, but it may not be adequate for silo discharge during which the coefficient of wall friction may markedly vary due to a relatively rapid sliding velocity of solid particles against the walls [9–11] or a slip–stick phenomenon [12]. A modification factor is usually adopted depending on the variability of the coefficient of wall friction, according to a normal wall pressure distribution [13,14]. Therefore, an

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investigation into how the wall pressure in a silo would be influenced by a varying wall friction condition is meaningful.

It is generally known that the coefficient of friction that resists the initiation of slipping from a sticking condition is different from the coefficient of friction that opposes on-going slipping. The former is typically referred to as the static coefficient of friction $\mu_s$, and the latter is referred to as the kinetic coefficient of friction $\mu_k$. Typically, the static coefficient of friction is higher than the kinetic coefficient of friction [15,16]. The static coefficient of friction is considered to correspond to the value given at zero sliding velocity, whilst a kinetic coefficient of friction corresponds to the value given at a high sliding velocity. The transition between static and kinetic friction is defined by a series of values specified at intermediate sliding velocities. Such a consideration is supported by a number of experimental observations and empirical formulas, which indicate that the coefficients of friction can be functions of sliding velocity between two surfaces in contact with each other [9,17]. Similarly, several empirical expressions have been proposed for quantitative study of the stationary contacting time effect on the coefficient of friction [16,18–21]. A wall friction model based on sliding velocity is applied in this study to evaluate the effect of varying wall friction on the wall pressure during silo discharge.

This paper presents the simulation of filling and discharge pressure in a cylindrical silo with a conical hopper using the FE model with Arbitrary Lagrangian–Eulerian formulation. The silo configuration is based on a model silo tested by the researchers at the Technical University of Demark [22], so the FE results can be compared with the experimental observations. The key aspects in the numerical modelling include dealing with the complex geometry of the silo and the wall boundary condition between the granular solid and the silo walls. The predicted dynamic pressures are studied in detail, and an interpretation of the fluctuating pressure patterns, which were also observed in the experiments, is provided in relation to the consequence of wave propagation and macro-slip motion.

## 2. Description of the model silo experiments and FE modelling considerations

To facilitate the development and verification of the FE model, the silo-hopper chosen for this study was based on the careful experiments conducted on a model silo [22]. This section provides a brief description of the experiments, followed by an outline of the general FE modelling considerations. Modelling of the silo transition is a significant topic, which will be treated separately in Section 3.

### 2.1. Brief description of the model silo tests

The model silo tests (herein called “Danish tests”) were carried out by Munch-Andersen and his co-workers at the Technical University of Demark using different granular materials [22]. The present study relates to the tests using dry cohesionless sand. The silo model consisted of cylindrical section with a steep conical hopper (see Fig. 1). The cylindrical section was 5 m high with an internal diameter of 0.70 m. The walls were made of epoxy with a thickness of 20 mm. The conical hopper, which was made of mild steel, had a circular outlet of a diameter 0.06 m and hence was designed with an apex half-angle $\alpha$ of 20°. A series of tests were carried out with different test conditions including smooth or rough walls and flat or hopper bottom. For this study, only the tests with smooth walls and a hopper bottom are considered.

### 2.2. FE modelling considerations

Finite element modelling of granular material during silo discharge requires appropriate handling of large deformations. To avoid mesh distortion during granular flowing, the FE simulations are performed using the uncoupled Arbitrary Lagrangian–Eulerian (ALE) formulation with an adaptive meshing technique in the Abaqus/Explicit code [23]. In the ALE formulation, the sides and top surface of the stored solid are treated as Lagrangian boundaries whilst the base (outlet) is set to be an Eulerian boundary. The adaptive meshing technique helps to control mesh distortion and maintain a high-quality mesh throughout the computation even though large deformation occurs.

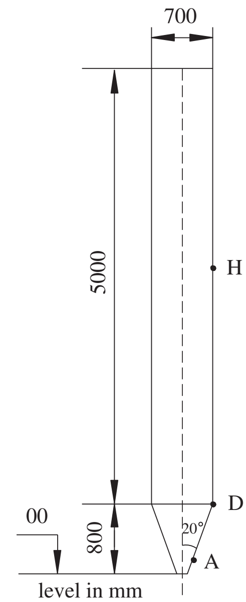


**Fig. 1.** Configuration of the Danish model silo: a cylindrical bin with a conical hopper.

The silo was assumed to be axisymmetric. The stored material was discretised using about 20,000 first-order 4-node quadrilateral elements (1 cm × 1 cm for element size) with reduced integration. The walls were modelled using about 300 non-deformable 2-node rigid elements. The stored material was modelled as elastic-perfectly plastic with a linear Drucker–Prager failure criterion [24]. A liner Drucker–Prager plastic flow rule was used with the Drucker–Prager failure criterion. Since a dilatancy angle of 10° was adopted in Abaqus, it is a non-associated flow [23]. The solid–wall interaction was first modelled using Coulomb stick–slip contact with a constant coefficient of wall friction.

Table 1 summarises the material properties used in the FE simulation. These properties came primarily from the Danish silo tests. The Young's modulus and Poisson's ratio, however, were not available from the tests and instead they were adopted from other FE simulations for dry and cohesionless sand [25]. It should be noted that the solid–wall friction coefficient was varied in order to examine the sensitivity of kinetic friction during silo discharge in Section 4. The internal friction angle of the material $\varphi_i$ was approximated by the repose angle of the sand measured in the Danish silo tests. This determination of the model parameter from the internal friction angle for the Drucker–Prager failure criterion gave an upper and a lower bound value of 51° and 45° respectively using the equations in Abaqus [23]. An average value, $\varphi_{DP} = 48°$, was used in the present FE simulation. A very small value of cohesion ($c = 1$ Pa) was also adopted to represent the cohesionless sand and to avoid numerical difficulties at near zero stress. A similar treatment, which was made by Ai et al. [26], has been proven

**Table 1**
Material parameters in FE model.

| | |
|---|---|
| Unit weight ($\gamma$) | 15.0 kN/m$^3$ |
| Young's modulus ($E$) | 2.0 MPa |
| Poisson's ratio ($\nu$) | 0.3 |
| Measured internal friction angle ($\phi_i$) | 40° |
| D-P internal friction angle ($\phi_{DP}$) | 48° |
| Dilatancy angle ($\psi$) | 10° |
| Cohesion ($c$) | 1 Pa |
| Coefficient of solid–wall friction | 0.67 |