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Flow characteristics and discharge rate of ellipsoidal particles in a flat bottom hopper

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ABSTRACT

Hopper flow characteristics are significantly affected by particle shape. In this work, ellipsoidal particles which can represent a large number of shapes are used to investigate the shape effect on granular flow in a cylindrical hopper. Numerical experiments are conducted by use of the discrete element method, with its validity verified by comparison with the results from physical experiments. The results indicate that particle shape can make a significant effect on the flow pattern. In particular, the increase of deviation from spheres can decrease the mixed region adjacent to the side wall, and increase the stagnant zone at the bottom corner. It may also lead to decreased wall stress. Furthermore, particle shape has a significant effect on the discharge rate. Spheres of unity aspect ratio have the highest flow rate, and the lower or higher aspect ratio, the smaller the flow rate. Based on the numerical results, the Beverloo equation is modified, where parameters *C* and *k* in the equation are respectively formulated as a function of aspect ratio.

1. Introduction

Hoppers are widely used in many industries such as mining, metallurgy, and food industries. To develop a comprehensive understanding of the dynamic behavior of granular flow in a hopper, extensive studies have been carried out by means of analytical, experimental and numerical approaches. In particular, discrete element method (DEM) plays an important role as it takes into account the discrete nature of granular materials without requiring any global assumption needed in the previous macroscopic approaches and thus allows a better understanding of the underlying mechanisms. As summarized by Zhu et al. [1], the studies of hopper flow by DEM mainly include wall stress/pressure, discharge rate and internal properties. In particular, prediction of the discharge rate is of importance for the design of reliable and controllable transport systems, and is difficult due to the complexity associated with granular flow such as inhomogeneous solid distribution, irregular velocity profile and diverse particle size and shape [2]. DEM simulations have been conducted by many investigators to predict discharge rate, showing a good agreement with experiments [3–5] and empirical predictions [6–8], particularly by the Beverloo equation [9]. Generally speaking, the Beverloo equation gives a good prediction for spherical particles. However, its predictability decreases with shape increasing deviation from spheres.

Many attempts have been made to examine the effect of particle shape on the discharge rate using DEM approach [10–19]. For example, Cleary [16] used the "super-particle" approach to identify the relationship between particle shape and hopper discharge rate, indicating a linear decrease in mass flow rate with increasing particle elongation. Langston et al. [13] found that elliptical particles of aspect ratio 5 are discharged 40% faster than circles, which was explained by particle alignment yielding a lower resistance to flow. But friction between particles was ignored in their work. Li et al. [14] studied the effect of inter-particle friction, and found that friction has little effect on the flow rate. But friction is reported to have a significant effect on discharge rate for elongated ellipsoids [15]. Fraige et al. [11] compared experimental results and DEM simulations for cubic and spherical particles, and demonstrated that cubes did not flow readily. This was ascribed to the geometric locking with cubes compared to spheres.

How particle shape affects discharge rate is not consistent in the above studies, influenced by model types (two dimensional or three dimensional), particle shapes (cubes, "super-particle", ellipse and ellipsoids), or particle properties (e.g. friction coefficients). The resulting understanding is fragmental without a clear picture on the effect of particle shape on the discharge rate. Also, the validity of the Beverloo equation to predict the discharge rate of ellipsoids, and its modification for ellipsoids and other shaped particles have not been addressed.

In this work, steady-state granular flow in a cylindrical hopper with a flat bottom is investigated by DEM. Ellipsoidal particles with a wide range of aspect ratios (from 0.3 to 3.0) are used in the simulation. The DEM model is firstly validated by comparison with physical

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experiments, then the effect of aspect ratio on flow patterns is discussed. Finally, the discharge rate for different aspect ratios is investigated, and the Beverloo equation is modified to take into account the effect of particle shape.

2. Model description

Techniques for DEM modeling of non-spherical particles have been reported in the literature [20]. In particular, a number of investigators have contributed to the modeling of ellipsoids [20–25]. The present DEM model is largely developed on this basis, and has been used to study packing [26] and gas fluidization of ellipsoids [27]. For completeness, a brief description of the DEM method used is given below.

A given particle in a granular system can have two types of motion: translational and rotational. During its movement, the particle may interact with its neighboring particles or walls, through which momentum and energy are exchanged. Newton's second law of motion is used to describe the motion of individual particles. Thus, at any time t, the equations governing the translational and rotational motion of particle i are respectively given as

$$m_i \frac{d\mathbf{v}_i}{dt} = \sum_{j=1}^{k_i} \left(\mathbf{f}_{c,ij} + \mathbf{f}_{d,ij} \right) + m_i \mathbf{g} \tag{1}$$

and

$$I_i \frac{d\boldsymbol{\omega}_i}{dt} = \sum_{j=1}^{k_i} \left(\mathbf{M}_{t,ij} + \mathbf{M}_{n,ij} + \mathbf{M}_{r,ij} \right) \tag{2}$$

where $\mathbf{v}_i$ and $\boldsymbol{\omega}_i$ are the translational and angular velocities of the particle, respectively, and k_i is the number of particles interacting with the particle. As shown in Fig. 1, the forces involved are: the gravitational force $m_i\mathbf{g}$, and inter-particle forces which include elastic force $\mathbf{f}_{c,ij}$ and viscous damping force $\mathbf{f}_{d,ij}$. The torques acting on particle i by particle j include: $\mathbf{M}_{t,ij}$ generated by the tangential force, $\mathbf{M}_{r,ij}$ commonly known as the rolling friction torque, and also the torque $\mathbf{M}_{n,ij}$ generated by the normal force when the normal force does not pass through the particle center.

Equations used to calculate the interaction forces and torques between two spheres have been well-established and widely used in the literature [28–30]. In this proposed model [26,27], the non-linear model for spheres is extended to ellipsoids and listed in Table 1. As noted above, an additional torque ($\mathbf{M}_{n,ij}$) is introduced, which results when the normal force does not pass through the center of an ellipsoid and, together with the tangential forces and rolling torque, governs the rotational motion of a particle. Another parameter is the so called reduced radius R^* in the calculation of the contact forces between particles i and j. For spheres, $R^* = R_iR_j / (R_i + R_j)$ where R_i and R_j are radii of particles i and j, respectively. But for ellipsoidal particles, $R^* = (A'B')^{-0.5} / 2$ where A' and B' are related to the radii of the particle shape curvature at a contact point. More details of the determination of A' and B' can be seen elsewhere, e.g. [31].

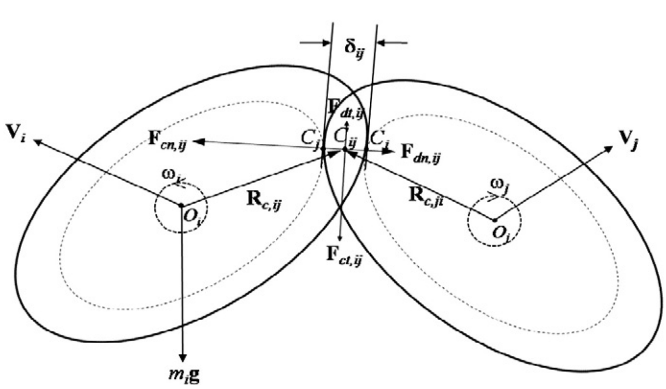

Fig. 1. Two-dimensional illustration of forces acting on ellipsoid i in contact with j.

Table 1
Components of forces and torques acting on ellipsoidal particle i.

Forces	Equations
Normal elastic force ($\mathbf{f}_{cn,ij}$)	$-4/3E^*\sqrt{R^*}\delta_n^{3/2}\mathrm{n}$
Normal damping force ($\mathbf{f}_{dn,ij}$)	$-c_n\left(8m_{ij}E^*\sqrt{R^*\delta_n}\right)^{1/2}\mathrm{v}_{n,ij}$
Tangential elastic force ($\mathbf{f}_{ct,ij}$)	$-\mu_s\lvert\mathbf{f}_{cn,ij}\rvert\left(1-(1-\delta_t/\delta_{t,\max})^{3/2}\right)\hat{\boldsymbol{\delta}}_t$
Tangential damping force ($\mathbf{f}_{dt,ij}$)	$-c_t\left(6\ \mu_s m_{ij}\lvert\mathbf{f}_{cn,ij}\rvert\sqrt{1-\delta_t/\delta_{t,\max}}/\delta_{t,\max}\right)^{1/2}\mathrm{v}_{t,ij}$
Coulumb friction force ($\mathbf{f}_{t,ij}$)	$-\mu_s\lvert\mathbf{f}_{cn,ij}\rvert\hat{\boldsymbol{\delta}}_t$
Torque by tangential forces ($\mathbf{M}_{t,ij}$)	$\mathbf{R}_{c,ij}\times(\mathbf{f}_{ct,ij}+\mathbf{f}_{dt,ij})$
Torque by normal force ($\mathbf{M}_{r,ij}$)	$\mathbf{R}_{c,ij}\times(\mathbf{f}_{cn,ij}+\mathbf{f}_{dn,ij})$
Rolling friction torque ($\mathbf{M}_{r,ij}$)	$\mu_{r,ij}\lvert\mathbf{f}_{n,ij}\rvert\boldsymbol{\omega}_{ij}^n$

Where $1/m_{ij} = 1/m_i + 1/m_j$, $E^* = E/2(1-\nu^2)$, $\boldsymbol{\omega}_{ij}^n = \boldsymbol{\omega}_{ij}^n/\left|\boldsymbol{\omega}_{ij}^n\right|$, $\hat{\boldsymbol{\delta}}_t = \boldsymbol{\delta}_t/\lvert\boldsymbol{\delta}_t\rvert$, $\delta_{t,\max} = \mu_s(2-\nu)/2(1-\nu)\cdot\delta_n$, $\mathbf{v}_{ij} = \mathbf{v}_j - \mathbf{v}_i + \boldsymbol{\omega}_j\times\mathbf{R}_{c,ji} - \boldsymbol{\omega}_i\times\mathbf{R}_{c,ij}$, $\mathbf{v}_{n,ij} = (\mathbf{v}_{ij}\cdot\mathbf{n})\cdot\mathbf{n}$, $\mathbf{v}_{t,ij} = (\mathbf{v}_{ij}\times\mathbf{n})\times\mathbf{n}$. Note that tangential forces ($\mathbf{f}_{ct,ij}+\mathbf{f}_{dt,ij}$) should be replaced by $\mathbf{f}_{t,ij}$ when $\delta_t \geq \delta_{t,\max}$.

The explicit time integration method is widely used to solve the translational and rotational motions of a system of discrete particles in DEM simulations [32]. Although it is established for spheres, such a method can be extended to ellipsoids. The difficulties associated with the extension mainly lie in two aspects: particle–particle detection and particle orientation, which are briefly given below.

(i) *Particle–particle contact detection*. The detection of particle contacts for ellipsoids is much more complicated than spheres. Various analytical methods are available to detect the contacts between ellipsoids, e.g. intersection algorithm [21], geometric potential algorithm [22–24], and common normal algorithm [23]. As the geometric potential algorithm is more reliable [20], it is hence used in the present model. In particular, the algorithm used in [23,24] is employed. Thus, as shown in Fig. 1, point C_i is defined as the "deepest" point of ellipsoid i penetrating into j in their collision, and can be determined through a numerical procedure detailed in [23,24]. The contact point between ellipsoids i and j is defined as the midpoint of the line connecting C_i and C_j, and then the normal and tangent planes are defined on this basis. The distance between the two special points is regarded as the particle overlap δ_{ij} which will be used to calculate the magnitude of normal contact force between particles i and j. It should be pointed out that the computational time for contact detection is huge. This is largely because the algorithm used to determine one contact point involves the numerical solution of a sixth-order polynomial equation [23].

(ii) *Particle orientation*. Particle orientation is another parameter that must be considered for non-spherical particles. It is generally described by three Euler angles (ϕ,θ,ψ) [33]. Briefly, at each time step, for the convenience of determining the inertia tensor $\mathbf{I}_i$ of an ellipsoid, the rotational equation expressed by Eq. (2) in the space-fixed coordinate system (x,y,z) is converted to the body-fixed coordinate system (x',y',z') which is a moving Cartesian coordinate system fixed with the particle and whose axes are superposed by the principal axes of inertia. Thus, in this converted coordinate system, the angular velocities $\boldsymbol{\omega}'_i$ of particles can be calculated as used for spheres; they are then used to determine the new three Euler angles on the basis of the so-called quaternion method. More details about the method can be found elsewhere (see, for example, [33]).

3. Simulation conditions

A packing is formed first in a cylindrical hopper with the diameter of 0.3 m. An orifice is located in the central flat bottom with its diameter