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Discharge rates of dry granular material from bins with lateral exit holes

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ABSTRACT

In this work we have analyzed experimentally the mass flow rate $\dot{m}$ of the lateral outflow of cohesionless sand beach and granulated sugar through circular orifices of diameter D and rectangular and triangular slots of hydraulic diameter D_H made in vertical walls of bins. Such experiments were also performed in order to determine the influence of the wall thickness of the bin, w. Geometrical and physical arguments are given to get a general correlation for $\dot{m}$ embracing both quantities, D (or D_H) and w. The angle of repose is also an important factor characterizing these gravity flows.

1. Introduction

The discharge of non cohesive granular materials, from the bottom exits of silos and bins, due to gravity action alone, is a very interesting phenomenon that involves dynamical events as jamming [1], density fluctuations [2] and industrial controlled flows such as dosage of powders and granules [3], among others.

The fundamental study of the law that obeys the mass flow rate of grains, $\dot{m}$ owed to Hagen [4] who, by 1852, performed experiments with sand and he found that $\dot{m}{\sim}\rho g^{1/2}D^{5/2}$, where ρ is the bulk density, g is the acceleration due to gravity, and D is the diameter of the circular orifice. Thus, the level of filling is not important. This result contrasts with the one occurring in liquids where the mass flow rate depends essentially on the level of filling above the discharge orifice. Actually, in specialized literature other correlations for discharge rates from silos with small openings [5] and hoppers filled with fine and coarse non-cohesive granular materials [6] have been reported.

Moreover, in their seminal paper on the flow of granular solids through circular orifices and slots Beverloo, Leniger and van de Velde [7] have reported other correlations for $\dot{m}$. Despite that, they found that the most suitable correlation to predict the mass flow rates from bottom exits in open-top bins, silos and hoppers is the so called Hagen–Beverloo correlation [7], which has the form $\dot{m}{\sim}\rho g^{1/2}(D-kd_g)^{5/2}$ where d_g is the mean diameter of the grain and k is a dimensionless constant with typical values k ~ 1–2 [8]. Hence, if $D/d_g \gg 1$ the Hagen formula predicts the discharge rates pretty well.

A very important observation is that several authors studied the discharge rates through slots with triangular and rectangular cross-sections [7,9]. In such case the correlation that best fits the experimental data has the form $\dot{m}{\sim}\rho g^{1/2}(D_H-kd_g)^{5/2}$ where D_H is the hydraulic diameter ($D_H =$ four times the area of the aperture divided by the perimeter) [7]; the concept of hydraulic diameter has been inherited from fluid mechanics [10].

Despite the enormous utility of Hagen's law (or its generalization, the Hagen–Beverloo law), only a few studies have been conducted to test its validity in the discharge of grains through orifices in the vertical walls of bins [11–17]. The aim of this work is to study experimentally the discharge rates of granular solids through vertical walls of open-top bins for different orifice sizes, D, different cross-sections of orifices and several wall thicknesses, w. As it will be seen later, the thickness of the wall can be used to control the discharge rate and even the granular flow can be arrested if w exceeds a critical value that depends also on the angle of repose of the granular material. Consequently, the term *wall thickness* will be used here assuming that the condition $w/d_g \gg 1$ is fulfilled, *i.e.*, when the average grain size is very small in comparison with w and, at the same time, the angle of repose (the steepest angle of descent of the slope relative to the horizontal plane when material on the slope face is on the verge of sliding) is well defined.

The plan of this work is as follows. Firstly, in the next section we review experimental studies of the mass flow rate from orifices in lateral walls of bins. Then, in Section 3, we report experiments of discharge rates from bottom and lateral exit holes, where the influence of D, D_H and w is examined. Then we propose, on the basis of our experimental results, a correlation that embraces both changes in D (or D_H)

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and w, in the case of lateral holes, and finally, in Section 4, we give the main conclusions of the study here tackled.

2. Background

To the best of our knowledge, Bagrintsev and Koshkovskii [11] were the first researchers who studied experimentally the problem of the gravity driven lateral outflow of granular material in cylindrical bins with vertical walls. They used oval and circular exit holes made in transparent plastic walls, and observed that "the outflow capacity decreases as wall thickness increases". Later on, experiments in silos with rectangular exit holes [12–14] and circular exit holes [13,15,16] were reported. In summary, these and other authors found that $\dot{m}{\sim}\rho g^{1/2}D^n$, with $n = 3.4$ [11], 3.3 [15,16], $2.6 < n < 3$ [13] and 2.5 [12,17]. It is important to note that in some studies [11–13], D is essentially the hydraulic diameter. Nevertheless, none of the referred works have analyzed systematically the effect of the wall thickness on $\dot{m}$.

3. Experiments

3.1. Bottom holes

In the present work, the simultaneous effects of the wall thickness w and the orifice diameter D on the mass flow rate $\dot{m}$ are examined. At a first glance it is easy to conclude that if the vertical wall is very thick, there will be no efflux of granular material.

On the other hand it is well known that for granular solids the wall thickness does not affect substantially the value of $\dot{m}_0$, the mass flow rate when the exit hole is located at the bottom of a silo (the opposite is common for porous aeratable powders, where aeration develops flow-obstructions near the exit holes [18,19]). In a first instance, we did experiments to measure $\dot{m}_0$ with sand beach (composed of irregular grains of mean diameter $d_g = 0.03$ cm, bulk density $\rho = 1.5 \pm 0.01$ g/cm^3 and angle of repose $\theta_r = 33° \pm 0.5° = 0.57 \pm 0.008$ rad; incidentally, Hagen [4] reported the same value for the bulk density) and granulated sugar (composed of grains of crystal-like shapes of mean diameter $d_g = 0.073$ cm, bulk density $\rho = 0.84 \pm 0.01$ g/cm^3 and angle of repose $\theta_r = 33.5° \pm 0.5° = 0.58 \pm 0.008$ rad; more mechanical properties could be found, for instance, in [20]) that crosses through circular orifices of diameters $D = 0.6$, 0.7, 0.8 and 1 cm and rectangular and triangular slots in an acrylic-made box with a thin wall and a thick wall. Dry sand and granulated sugar are familiar examples of free-flowing bulk solids and this was the main reason to use these materials in our work. The section of the laboratory in which the experiments were done was climate controlled (25 ± 1 °C and 45 ± 10% R.H.). The moisture contents of sand and granulated sugar samples were 0.50 ± 0.06% and 0.015 ± 0.005% w.b., respectively.

In Fig. 1 we show pictures of the samples of sand (Fig. 1(a)) and granulated sugar (Fig. 1(b)) used in our experiments. Pictures were obtained using a Steindorff digital microscope and the corresponding particle size distributions (Fig. 2) were determined using the microscope software which allowed us to find the surface area of each particle and its surface diameter. This method yields the average (median) particle sizes which were 0.03 cm for sand and 0.073 cm for sugar.

Experiments were made upon a transparent box, 10 × 10 cm^2 inner cross-section and 50 cm height. In experiments with bottom exits we have used sand and granulated sugar in bins with several bottom wall thicknesses $w = 0.3$ cm and $w = 0.9$ cm. Details of the measurement procedure of the discharge rates are given afterwards. In Fig. 3 we show the experimental plots of $\dot{m}_0$, as a function of $\rho g^{1/2}D^{5/2}$, for both materials. In this figure we observe that both cases fit very well the Hagen's law (straight lines), and thus the effect of w is not observed. Hence, the relation that fits the experimental data has the form

$$\dot{m}_0 = a\rho g^{1/2}D^{5/2}, \tag{1}$$

where the dimensionless discharge coefficient has the value $a = 0.48$ for sand and $a = 0.46$ for sugar. Here it is important to comment that it has been reported that a may be dependent on the coefficient of friction of the granular material, μ [5,21], on the geometrical characteristics of the silo [5,21], on the grain shapes [22] and on the orifice shape [13]. So far, a may be different for different materials.

In a second series of experiments we have made measurements of the discharge rates of sand and sugar across four horizontal slots with cross-sections having equilateral triangular shapes with lengths $l = 1$, 1.5, 2 and 2.5 cm, respectively. The hydraulic diameters of these slots were computed by using the relation $D_H = l/\sqrt{3}$ [7,10]. Similarly, four slots with rectangular shapes were made, $e = 1$ cm width and $f = 1.5$, 2, 2.5 and 3 cm heights, in these cases $D_H = 2(ef)/(e + f)$. All these slots were made on the bin walls with a thickness of 1.2 and 1.8 cm, respectively. In Fig. 4 we show plots of the mass flow rates for both types of slots, as a function of $\rho g^{1/2}D_H^{5/2}$. It is easily appreciated that the best fit obeys the relation

$$\dot{m}_0 = a_H\rho g^{1/2}D_H^{5/2}, \tag{2}$$

where for bins filled with sand and slots with rectangular cross-sections the slope yields $a_H = 0.89$ and for triangular cross-sections the slope is

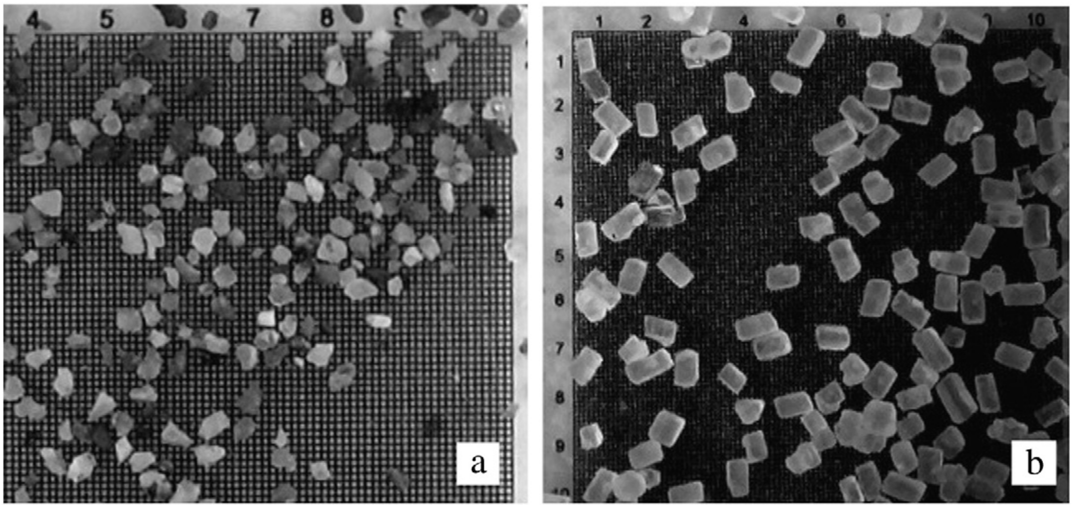

Fig. 1. Micrographs of the particle shapes of samples of sand (a) and granulated sugar (b) used in our experiments. Distance among numbers is 1 mm.