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Explicit formulation for suspended concentration distribution with near-bed particle deficiency

Snehasis Kundu, Koeli Ghoshal *

Department of Mathematics, Indian Institute of Technology, Kharagpur 721302, India

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ABSTRACT

An explicit expression for concentration distribution in suspension in a turbulent flow with near-bed particle deficiency (i.e. where the maximum concentration appears at some distance above the bed, referred to as 'type II profile' in the present work) is proposed. Initially, an asymptotic solution is expressed in each of the region after dividing the entire flow depth into two regions: inner suspension region and outer suspension region. The final model is obtained by deploying Almedeij's asymptotic matching method. Efficiency of the model is tested for dilute and dense flow through pipes and open-channels. The model is compared with available experimental data as well as with other suitable models existing in the literature. Good agreement between the observed value and computed result, and minimum error indicate that the present model is more accurate in predicting particle concentration distribution for type II profile under different flow conditions. The important finding of this study is that particle lift depends on particle size, particle density, particle fluctuating intensity and volumetric concentration. Results show that type II profile may occur for both sand and gravel particles. In addition, it is shown that in dense turbulent flow the location of maximum concentration point depends on average volumetric concentration.

1. Introduction

Suspension of solid particles in a turbulent flow is an important topic of research and has wide applications in industry and geophysical research. The knowledge of particle concentration profile helps us to model sediment transport in river. Numerous investigations have been done with the problem of particle suspension concentration distribution in two-dimensional steady turbulent flow. O'Brine [1] and Rouse [2] were pioneers in this field to analyze the mechanism of particle suspension in turbulent flow using diffusion theory. Diffusion theory indicates that the rate of upward transfer of suspended particles due to turbulent diffusion is balanced by downward settlement due to gravitational force and it is considered as 'mirror' effect as mentioned by [3]. The description based on diffusion theory physically implicates that only gravity, vertical drag and buoyancy effects are considered whereas particle inertia, fluid–particle interactions and particle–particle interactions are not taken into account. Besides diffusion theory, mixing length theory [4–7], gravitational theory [8], similarity theory [9] and stochastic theory [10] have been used to describe the particle suspension. All the mentioned theories consider the transport of solid particles in turbulent flow as a one-phase system but in real situations two separate phases are present. Recent advances in sediment transport through turbulent flow reckon particle-fluid mixture as a solid/liquid two-phase system. The solid/liquid two-phase system can be described either by microscopic method (kinetic theory) or by macroscopic method (continuum theory). Ni and Wang [11] demonstrated that the continuum theory underpins most previous theories including diffusion theory, mixing theory and gravitational theory. Besides diffusion theory, although aforementioned theories are remarkable but do not overcome the limitations of diffusion theory [11]. In recent years, different researchers have adopted two-phase flow theory to describe sediment-laden open channel flow [12–20]. The two-phase flow theory considers that both phases (sediment and water in this study) are continuous medium and obey basic conservation laws. The physics of this theory helps to explain the effects of sediment inertia, sediment–turbulent interactions but sediment–sediment interactions are usually ignored. A potentially more accurate description of the sediment suspension process can be performed by using kinetic theory. In this approach, once the sediment probability density function (PDF) is known, other macroscopic components such as sediment concentration distribution can be derived and all statistical information of sediment motions are subsequently obtained. In this standpoint, kinetic theory overcomes the shortcomings of the two-phase theory as it characterizes sediment–sediment interactions. Many researchers applied kinetic theory to describe particle suspension distribution in turbulent flow and found that it properly describes type II profile of particle suspension distribution.

The existence of two types of concentration profiles was experimentally observed both in pipe flow and open-channel flow [21–25]. Type I profile is defined as the concentration profile where the maximum concentration appears at the bottom very near to the bed whereas type II

* Corresponding author.

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profile is defined as the concentration profile where the maximum concentration appears at a height which is significantly above the bed. Though the application of continuum theory in solid/liquid system is encouraging, it fails to give a proper interpretation of certain experimental results. For example, type II profile cannot be explained using continuum theory. Ni and Wang [21] proposed that two types of concentration distribution can be explained from the fluctuating characteristics of fluid and particles. Similar analogy with the motion of gas molecules, Wang and Ni [25,26] and Ni et al. [24] used kinetic theory incorporating the effect of fluid induced lift force on concentration distribution and proposed model to describe concentration distribution. Recently, Fu et al. [27] developed a kinetic theory model for sediment-laden turbulent flow. This model accounts for the effects of sediment–turbulent interactions as well as sediment–sediment collisions. They also derived mathematical model for particle concentration distribution and their analysis shows that fluid induced lift force and granular temperature (measure of the strength of particle velocity fluctuation) are effective factors for type II distribution of concentration. Wang et al. [28] performed kinetic-model-based simulation incorporating the effects of fluid–sediment and sediment–sediment interactions as well as sediment inertia to analyze nonmonotonic concentration profile. In their analysis they found that for suspended sediments with large size or inertia, a noticeable velocity lag exists between sediments and that of carrier water and that sediment concentration profile shows a non-single valued distribution across the flow depth. Later on, Ni et al. [29] studied the characteristics of hyperconcentrated flow using kinetic theory. They observed that when average volumetric concentration $\overline{C}>0.4$ vertical profile of particle concentration alters and becomes unusual. Liu et al. [30] argued that particle concentration profile is significantly influenced by particle fluctuation intensity and flow uplift force.

Though the mechanism of particle suspension in flow is mainly supported by turbulent diffusion, there is distinction between the mechanism of suspension in the core flow region and in the near bed region. In the core flow region, suspension is supported by ‘mirror’ effect but in the near bed region due to diminished turbulence, particles are not lifted up by the diffusion process and the second contribution to suspension, hydraulic lift, comes into prominence [3]. Numerous research results show that hydraulic lift force on particles by ambient fluid (or ‘off-the-wall’ lift force [3] or comprehensive lift force according to [31]), granular temperature, particle–particle interaction (important in dense flow) and viscous-turbulent interface effect are important factors for particle suspension in the near bed region [21,25,30,32]. Various theoretical and experimental investigations show that light (particles having a smaller density compared to surrounding fluid) coarser particles have more tendency to move upward as the lift force and viscous turbulent interface have significant effects on large particles [25,27,33]; consequently the maximum concentration point is shifted upward. As wide research has been done for type I distribution, this study focuses only the case of type II distribution of particle concentration.

In engineering applications, besides analytical and numerical approach, observational approach is employed for data modeling technique. Each of these approaches is powerful and flexible and can be applied to different problems. In this study, to describe the type II profile of suspension concentration distribution observational approach is considered. Almedeij [34] proposed an asymptotic matching approach to model environmental problems with complicated behavior. The main objectives of this study are: (i) to develop a model for type II concentration distribution using Almedeij's asymptotic matching method; (ii) to check the validity of the proposed model for a wide range of experimental data (dilute and dense flow through pipes and open-channels); and (iii) to compare the model with previous models.

2. Approach using Almedeij's asymptotic matching method

Almedeij [34] proposed an approach for modeling observed data by dividing the entire range of data into smaller segments and an overall relationship is obtained combining these segments. An empirical model is developed to describe the type II profile of concentration distribution in dilute and dense flow using this asymptotic matching method.

In type II profile, sediment concentration initially increases with characteristic height ξ ($=y\,/\,h$, where y is the vertical coordinate and h is the flow depth) in the region $\xi_a \le \xi \le \xi_{max}$ (see Fig. 1), and then begins to decrease when characteristic height ξ exceeds the critical height ξ_{max} (where characteristic height ξ_{max} corresponds to maximum volumetric concentration) in the region $\xi_{max} \le \xi \le 1$. Here ξ_a denotes the reference level for suspension concentration distribution and is considered as the lowest level in suspension. This nonmonotonic behavior of suspension distribution indicates that the entire flow depth can be divided into two regions: inner suspension region where sediment concentration increases with characteristic height from sediment bed and $\xi_a \le \xi \le \xi_{max}$; and outer suspension region above the inner suspension region where sediment concentration decreases with characteristic height ξ and $\xi_{max} \le \xi \le 1$. For each of the region, the relation of sediment concentration with characteristic height ξ from sediment bed can be expressed as

$$\frac{C}{\overline{C}} = b\xi^{\alpha} + a. \tag{1}$$

The best fitting constants a and b and the exponent α for each of the region can be determined by using the least-square technique. In this context, it is worthwhile to mention that suspension concentration distribution is usually described by a power law [35,36] which can be obtained from Eq. (1) if one considers $a = 0$. Under certain flow conditions, suspended concentration distribution in inner and outer suspension region can be approximated by linear profiles (e.g. pipe data of Michalik, Fig. 3 of [23]. For such situations, concentration profile in each of the region can be described by Eq. (1) with $\alpha = 1$ which indicates the advantage of using Eq. (1). A simple example of power law profile is shown in Fig. 1 where it can be observed that in inner suspension region and outer suspension region sediment concentration can be represented by two power laws with monotonically decreasing exponent α. Similarly for linear profile also, sediment concentration profiles in inner suspension region and outer suspension region are expressed by two linear segments with monotonically decreasing slope b.

Following this, functions φ_1 and φ_2 for inner and outer suspension region can be expressed respectively as

$$\varphi_1 = \left(b_1\xi^{\alpha_1} + a_1\right)^m \quad \text{where} \quad \xi_a \le \xi \le \xi_{max} \tag{2}$$

and

$$\varphi_2 = \left(b_2\xi^{\alpha_2} + a_2\right)^m \quad \text{where} \quad \xi_{max} \le \xi \le 1 \tag{3}$$

where either $a_1 = a_2 = 0$ (for power law) or $\alpha_1 = \alpha_2 = 1$ (for linear profile) and m is a constant determined by minimizing the error of the model. For power law $\alpha_1 > \alpha_2$ and for linear profile $b_1 > b_2$.

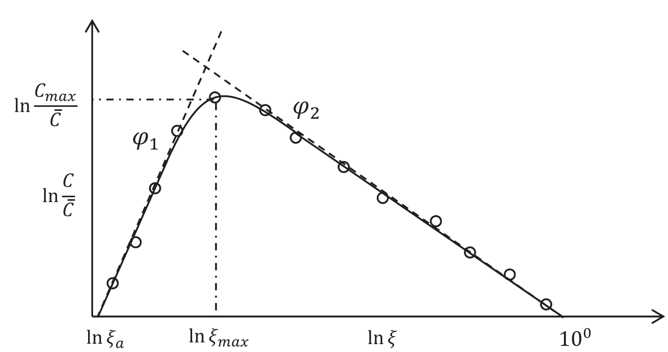

Fig. 1. Schematic diagram of the model.