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Short communication

Transient vertically motion of a soluble particle in a Newtonian fluid media

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ABSTRACT

Unsteady settling behavior of a soluble spherical particle falling in a Newtonian fluid media is investigated using a drag coefficient of the form given by Ferreira et al. (Chem. Eng. Commun. 1998). It is considered that the mass of the particle reduces due to its solubility in the fluid, and consequently diameter of the particle will be reduced by a linear function. In a current study, the equation of the motion for described variable-mass particle is introduced for the first time and is solved by Padé approximation of Differential Transformation Method (DTM-Padé) and numerical Runge–Kutta method. Also the influence of solubility parameter on velocity profile is discussed and particle's positions are depicted graphically in each 1 s time step.

1. Introduction

Description of the motion of immersed bodies in fluids has long been a subject of great interest due to its wide range of applications in industry e.g. sediment transport, deposition in pipelines, and alluvial channels [1,2]. The settling mechanism of solid particle, bubble or drop in Newtonian and non-Newtonian fluids is reported by Clift et al. [3] and Chhabra [4]. Several types of drag coefficients for spherical and non-spherical particles were presented by Haider and Levenspiel [5], also Guo [6] and Mohazzabi [7] studied the behavior of spheres and objects falling into fluids. Reviewing the technical literature, it is clear that most of the pervious investigations are performed for steady-state conditions (at terminal velocity) and few of them has been studied the unsteady motion of falling objects (accelerating motion). Also, none of them considered the reduction of the mass and diameter of the particle due to solubility or reaction in the fluid media, and they just considered a solid particle with a constant mass and diameter. In this study we aim to fill this gap with a novel analytical method.

Recently, several attempts have been made to develop analytical tools to solve the motion's equation of the falling objects in fluids. Ganji [8] employed Variational Iteration Method (VIM) and derived a semi-exact solution for the instantaneous velocity of the particle over time in incompressible fluids. Also, Yaghoobi and Torabi [9] investigated the acceleration motion of a vertically falling non-spherical particle in incompressible Newtonian media by VIM. Jalaal et al. [10] used Homotopy Analysis Method (HAM) for obtaining the solution of the one-dimensional non-linear particle equation. They demonstrated that using appropriate initial guess and auxiliary parameter, HAM is an accurate and reliable method. Furthermore, Jalaal et al. [11] applied He's Homotopy Perturbation Method (HPM) to solve the acceleration motion of a vertically falling spherical particle in incompressible Newtonian media. Torabi and Yaghoobi [12] combined HPM with Padé approximation for increasing the solution accuracy of the particle's equation of motion. The motion of a spherical particle rolling down an inclined plane submerged in a Newtonian environment has been studied by Jalaal et al. [13,14] through HPM.

In most applications of particles falling in liquids such as mineral processing, hydraulic transport, slurry systems, abrasive water jets, fluidized bed reactors and so on [3], the mass and consequently diameter of the particle reduce due to its solubility or reaction with fluid, while all of the previous works are based on a rigid and insoluble sphere particle. So in this paper, unsteady motion of a soluble particle in Newtonian media is investigated by Differential Transformation Method (DTM) [15] with Padé approximation and fourth order Runge–Kutta numerical method.

2. Problem description

For modeling the particle sediment phenomenon, consider a small particle with a spherical shape of variable diameter $D(t)$ and mass of $m(t)$ and density of ρ_s, falling in infinite extent filled by an incompressible Newtonian fluid. Density of fluid, ρ, and its viscosity, μ, are known. We considered the gravity, buoyancy, drag forces and added mass (virtual mass) effect on particle. According to the Basset–Boussinesq–Ossen (BBO) equation for the unsteady motion of the particle in a fluid, for a dense particle falling in light fluids and by assuming $\rho \ll \rho_s$, Basset

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Nomenclature

C_D	Drag coefficient
D_0	Initial particle diameter
$\dot{D}$	Rate of diameter reduction
$D(t)$	Particle diameter function
DTM	Differential Transformation Method
$f(\eta)$	Analytic function
g	Acceleration due to gravity [m/s^2]
H	Constant value
$m(t)$	Particle mass function
Num	Numerical method
P	Padé approximation
t	Time [s]
u	Velocity [m/s]
$x(t)$	Analytic function
$X(k)$	DTM transformed function
$[L,M]$	Order of accuracy in Padé

Greek symbols

μ	Dynamic viscosity [kg/ms]
ρ	Fluid density [kg/m^3]
ρ_s	Spherical particle density [kg/m^3]

history force is negligible. So, by rewriting force balance for the particle, the equation of motion is gained as follows [12],

$$m(t)\frac{du(t)}{dt} = m(t)g\left(1-\frac{\rho}{\rho_s}\right)-\frac{1}{8}\pi D(t)^2\rho C_D u(t)^2-\frac{1}{12}\pi D(t)^3\rho\frac{du(t)}{dt}, \quad (1)$$

where C_D is the drag coefficient, in the right hand side of Eq. (1), the first term represents the buoyancy effect, the second term corresponds to drag resistance, and the last term is due to the added mass effect which is due to acceleration of fluid around the particle. The main difficulty of solving Eq. (1) is non-linear terms due to the non-linearity nature of the drag force which comes from drag coefficient C_D, $u(t)^2$ and $D(t)^2$ terms. Ferreira et al. [16], in their analytical study, suggested a correlation for C_D of spherical particles which has good agreement with the experimental data in a wide range of Reynolds number, $0 \le \text{Re} \le 10^5$. This appropriate equation is

$$C_D = \frac{24}{\text{Re}}\left(1+\frac{1}{48}\text{Re}\right). \quad (2)$$

It's necessary to inform that Eq. (2) is based on the non-slip condition on the surface of the soluble particle. Jalaal et al. [11] have shown that Eq. (2) represents a more accurate resistance of the particle in comparison with the pervious equations presented by others. Substituting Eq. (2) into Eq. (1) and variable-mass of the spherical particle

$$m(t) = \frac{1}{6}\pi D(t)^3\rho_s \quad (3)$$

Eq. (1) can be rewritten as

$$\frac{1}{12}\pi D(t)^3(2\rho_s+\rho)\frac{du(t)}{dt}+3\pi D(t)\mu u(t) + \frac{1}{16}\pi D(t)^2\rho u(t)^2-\frac{1}{6}\pi D(t)^3 g(\rho_s-\rho) = 0. \quad (4)$$

It is completely evident that in different industrial processes and applications, diameter of the particle varies by a known function which depends onto its solubility. In this study, it is considered that diameter varies through a linear function, so for other functions it can be solved easily too by the same method.

$$D(t) = D_0 - \dot{D}\cdot t, \quad (5)$$

where D_0 is the initial diameter and $\dot{D}$ is the reduction rate of the diameter due to particle solubility. By substituting to Eq. (4),

$$\frac{1}{12}\pi(D_0-\dot{D}\cdot t)^3(2\rho_s+\rho)\frac{du(t)}{dt}+3\pi(D_0-\dot{D}\cdot t)\mu u(t) + \frac{1}{16}\pi(D_0-\dot{D}\cdot t)^2\rho u(t)^2-\frac{1}{6}\pi(D_0-\dot{D}\cdot t)^3 g(\rho_s-\rho) = 0. \quad (6)$$

Eq. (6) is a non-linear equation with an initial condition ($u(0) = 0$) which can be solved by numerical and analytical methods. In the present study, DTM-Padé and Runge–Kutta methods are presented for solving the problem. A schematic of described problem is shown in Fig. 1.

3. Differential Transformation Method with Padé approximate (DTM-Padé)

For understanding the method's concept, suppose that $x(t)$ is an analytic function in domain D, and $t = t_i$ represents any point in the domain. The function $x(t)$ is then represented by one power series whose center is located at t_i. The Taylor series expansion function of $x(t)$ is in the form of:

$$x(t) = \sum_{k=0}^{\infty}\frac{(t-t_i)^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=t_i} \quad \forall t \in D. \quad (7)$$

The Maclaurin series of $x(t)$ can be obtained by taking $t_i = 0$ in Eq. (7) expressed as:

$$x(t) = \sum_{k=0}^{\infty}\frac{t^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=0} \quad \forall t \in D. \quad (8)$$

As explained in [15] the differential transformation of the function $x(t)$ is defined as follows:

$$X(k) = \sum_{k=0}^{\infty}\frac{H^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=0}, \quad (9)$$

where $X(k)$ represents the transformed function and $x(t)$ is the original function. The differential spectrum of $X(k)$ is confined within the interval $t \in [0,H]$, where H is a constant value. The differential inverse transform of $X(k)$ is defined as follows:

$$x(t) = \sum_{k=0}^{\infty}\left(\frac{t}{H}\right)^k X(k). \quad (10)$$

From Eq. (10), it can be carried out easily that the theory of differential transformation is based upon the Taylor series expansion. The values of function $X(k)$ at values of argument k are referred to as discrete, i.e. $X(0)$ is known as the zero discrete, $X(1)$ as the first discrete, etc. The more discrete available, the more precise it is possible to restore the unknown function. The function $x(t)$ consists of the T-function $X(k)$, and its value is given by the sum of the T-function with $(t/H)^k$ as its coefficient. In real applications, at the right choice of constant H, the larger values of argument k the discrete of spectrum reduce rapidly. The function $x(t)$ is expressed by a finite series and Eq. (10) can be written as:

$$x(t) = \sum_{k=0}^{n}\left(\frac{t}{H}\right)^k X(k). \quad (11)$$

Some important mathematical operations performed by the Differential Transform Method are listed in Table 1. Many advantages of