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Laminar flow and heat transfer of nanofluid between contracting and rotating disks by least square method

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ABSTRACT

In this study, asymmetric laminar flow and heat transfer of nanofluid between contracting rotating disks are investigated. The fluids in the channel are water containing different nanoparticles *Cu*, *Ag* and Al_2O_3. The effective thermal conductivity and viscosity of nanofluid are calculated by the Chon and Brinkman models, respectively. The governing equations are solved via the fourth-order Runge–Kutta–Fehlberg method (NUM) and least square method (LSM). The effects of the nanoparticle volume fraction, rotational Reynolds number, injection Reynolds number, expansion ratio and *s* on flow and heat transfer are considered. The results show that as *s* increases temperature profile increases and the point of maximum radial velocity is shifted towards the middle of the two disks. Also the results indicated that temperature profile becomes more flat near the middle of two disks with the increase of injection but opposite trend is observed with increase of expansion ratio.

1. Introduction

The incompressible fluid flow and heat transfer over rotating bodies have many industrial and engineering applications such as gas turbine engines and electronic devices having rotary parts and have been studied in many industrial, geothermal, geophysical, technological and engineering fields. Originally Von Kármán [1] discussed the steady flow of Newtonian fluid over a rotating disk, who introduced an elegant transformation that enabled the Navier–Stokes equations for an isothermal, impermeable rotating disk to be reduced to a system of coupled ordinary differential equations. Using momentum integral method, he obtained an approximate solution to the ordinary differential equations (ODEs). Hydromagnetic flow between two horizontal plates in a rotating system, where the lower plate is a stretching sheet and the upper is a porous solid plate, was analyzed by Sheikholeslami et al. [2]. They reported that increasing magnetic parameter or viscosity parameter leads to decreasing Nu, while with increasing the rotation parameter, blowing velocity parameter, and Pr, the Nusselt number increases. Sibanda and Makinde [3] investigated the hydromagnetic steady flow and heat transfer characteristics of an incompressible viscous electrically conducting fluid past a rotating disk in a porous medium with the ohmic heating and viscous dissipation, they found that magnetic field retards the fluid motion due to the opposing Lorentz force generated by the magnetic field and the magnetic field and Eckert number tend to enhance the heat transfer efficiency. The three-dimensional problem of nanofluid between parallel plates in presence of variable magnetic field is studied by Hatami and Ganji [4]. In low Prandtl numbers, Nusselt number changes are not sensible, while in greater Prandtl numbers, the changes in Nusselt number are more noticeable. Hossain et al. [5] and Maleque and Sattar [6] also investigated the influence of variable properties on the physical quantities of the single rotating disk problem by obtaining a self-similar solution of the Navier–Stokes equations along with the energy equation.

There are some simple and accurate approximation techniques for solving differential equations called the Weighted Residuals Methods (WRMs). Collocation, Galerkin and least squares are examples of the WRMs. Stern and Rasmussen [7] used collocation method for solving a third order linear differential equation. Recently least squares method is introduced by Aziz and Bouaziz [8] and is applied for a predicting the performance of a longitudinal fin [9]. They found that least squares method is simple compared with other analytical methods. Shaoqin and Huoyuan [10] developed and analyzed least-squares approximations for the incompressible magneto-hydrodynamic equations. Aplication of LSM in different engineering problems can be found in Refs. [11–15].

Fluid heating and cooling are important in many industry fields such as power, manufacturing and transportation. Effective cooling

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techniques are absolutely needed for cooling any sort of high energy device. Common heat transfer fluids such as water, ethylene glycol, and engine oil have limited heat transfer capabilities due to their low heat transfer properties. In contrast, metal–thermal conductivities are up to three times higher than the fluids, so it is naturally desirable to combine the two substances to produce a heat transfer medium that behaves like a fluid, but has the thermal conductivity of a metal. Recently, several studies are investigated about nanofluid. The problem of laminar nanofluid flow in a semi-porous channel in the presence of transverse magnetic field was investigated analytically by Sheikholeslami et al. [16]. Their results showed that velocity boundary layer thickness decreases with increase Reynolds number and it increases as Hartmann number increases. Soleimani et al. [17] studied natural convection heat transfer in a semi-annulus enclosure filled with nanofluid using the Control Volume based Finite Element Method. They found that the angle of turn has an important effect on the streamlines, isotherms and maximum or minimum values of local Nusselt number. Steady magnetohydrodynamic free convection boundary layer flow past a vertical semi-infinite flat plate embedded in water filled with a nanofluid has been theoretically studied by Hamad et al. [18]. They found that Cu and Ag nanoparticles proved to have the highest cooling performance for this problem. Heat transfer of a nanofluid flow which is squeezed between parallel plates was investigated analytically using the Homotopy perturbation method (HPM) by Sheikholeslami and Ganji [19]. They reported that the Nusselt number has a direct relationship with nanoparticle volume fraction, the squeeze number and Eckert number when two plates are separated but it has reverse relationship with the squeeze number when two plates are squeezed. Several recent studies on the modeling of nanofluid flow and heat transfer have been studied [20–40].

In this study, the purpose is to solve nonlinear equations of the laminar nanofluid between contacting and rotating plates through the LSM. The effect of active parameter on velocity and temperature profiles has been examined.

2. Formulation of the problem

We consider the motion of nanofluid between contracting or expanding, rotating disks. The top and bottom boundaries are porous and heated disks. The distance between the disks is $2a(t)$. The disks have different permeability and expand or contract uniformly at a time-dependent rate $\dot{a}(t)$. As shown in Fig. 1, a cylindrical coordinate

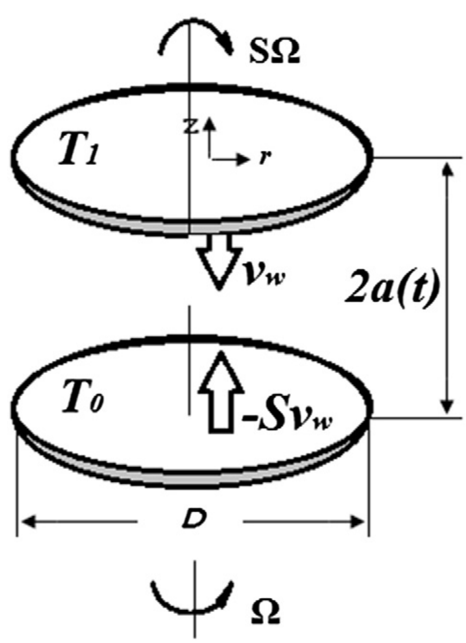

Fig. 1. Nanofluid flow between expanding and contracting porous disks.

system may be chosen with the origin at the middle of the disks. The velocity components u, v, w are taken to be in the r, ζ, z directions in this cylindrical coordinate system, respectively.

Under these assumptions, the continuity, and the momentum equations are given by the following relations, respectively [3].

$$\frac{1}{r}\frac{\partial(ru)}{\partial r}+\frac{1}{r}\frac{\partial v}{\partial \zeta}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{r}v\frac{\partial u}{\partial \zeta}+w\frac{\partial u}{\partial z}-\frac{1}{r}v^2=-\frac{1}{\rho_{nf}}\frac{\partial p}{\partial r}+\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u}{\partial \zeta^2}+\frac{\partial^2 u}{\partial z^2}-\frac{u}{r^2}-\frac{2}{r^2}\frac{\partial v}{\partial \zeta}\right) \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+\frac{1}{r}v\frac{\partial v}{\partial \zeta}+w\frac{\partial v}{\partial z}+\frac{1}{r}vu=-\frac{1}{\rho_{nf}r}\frac{\partial p}{\partial \zeta}+\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v}{\partial \zeta^2}+\frac{\partial^2 v}{\partial z^2}-\frac{v}{r^2}+\frac{2}{r^2}\frac{\partial u}{\partial \zeta}\right) \tag{3}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+\frac{1}{r}\upsilon\frac{\partial w}{\partial \zeta}+w\frac{\partial \upsilon}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\upsilon\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 w}{\partial \zeta^2}+\frac{\partial^2 w}{\partial z^2}\right). \tag{4}$$

The energy equation for an incompressible viscous fluid is

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial r}+\frac{1}{r}v\frac{\partial T}{\partial \zeta}+w\frac{\partial T}{\partial z}=\frac{k_{nf}}{\left(\rho C_p\right)_{nf}}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \zeta^2}+\frac{\partial^2 T}{\partial z^2}\right). \tag{5}$$

The effective density (ρ_{nf}), heat capacitance $(\rho C_p)_{nf}$ and viscosity of nanofluid (μ_{nf}) are defined as [41]:

$$\begin{aligned}
&\rho_{nf}=\rho_f(1-\phi)+\rho_s\phi\\
&\left(\rho C_p\right)_{nf}=\left(\rho C_p\right)_f(1-\phi)+\left(\rho C_p\right)_s\phi\\
&\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}}.
\end{aligned} \tag{6}$$

The effective thermal conductivity of the two component entities of spherical-particle suspension was introduced by Chon et al. [42] as follows:

$$\begin{aligned}
&\frac{k_{nf}}{k_f}=1+64.7\phi^{0.74}\left(\frac{d_f}{d_p}\right)^{0.369}\left(\frac{k_s}{k_f}\right)^{0.747}\mathrm{Pr}^{0.9955}\mathrm{Re}^{1.2321}\\
&\mathrm{Pr}=\frac{\mu_f(\rho C_P)_f}{\rho_f k_f},\quad \mathrm{Re}=\frac{\rho_f k_B T_w}{3\pi\mu_f^2\ell_f},\\
&\ell_f=0.17nm,\qquad k_B=1.3807\times10^{-23},\quad T_w=20^{\circ}C.
\end{aligned} \tag{7}$$

There are no slip and no temperature jump at the lower and upper disks. The boundary conditions are

$$\begin{aligned}
&u=0,\quad v=Sr\Omega,\quad w=v_w=A\dot{a},\quad T=T_1,\quad y=a(t)\\
&u=0,\quad v=r\Omega,\quad w=-sv_w=A_0\dot{a},\quad T=T_0,\quad y=-a(t)
\end{aligned} \tag{8}$$

where Ω is the angular velocity of the top disk, SΩ is the angular velocity of the upper disk, and $A_0=sv_w/\dot{a}=sA$ is the measure of wall permeability.