Powder Technology 235 (2013) 376–385

Contents lists available at SciVerse ScienceDirect

Powder Technology

journal homepage: www.elsevier.com/locate/powtec

Numerical simulation of particle-laden gas flow by Vortex in Cell method

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ARTICLE INFO

Article history:
Received 18 July 2012
Received in revised form 1 October 2012
Accepted 24 October 2012
Available online 31 October 2012

Keywords:
Gas–particle two-phase flow
CFD
Vortex method
Vorticity

ABSTRACT

This study proposes a simulation method for incompressible gas flow laden with small solid particles. It is based on a Vortex in Cell (VIC) method, which was originally developed to simulate incompressible single-phase flows. The proposed VIC method discretizes the gas vorticity field into vortex elements and computes the time evolution of the two-phase flow by calculating the behavior of the vortex element as well as the particle motion with the Lagrangian approach. This study also applies the VIC method to simulate a free fall of small solid particles in an unbounded air. The particles, initially arranged within a spherical region in a quiescent air, are made to fall, and their fall induces the air flow around them. The interactions between the particle motion and the air flow are favorably compared with the existing measured and simulated results, demonstrating the validity of the proposed VIC method.

1. Introduction

Gas flows containing small solid particles are found in many industrial applications, such as energy conversion systems and chemical reactors. They are also closely related with atmospheric phenomena, such as snowstorm and microburst. Various simulation methods have thus far been proposed for such particle-laden gas flow [1]. Most of the methods are of a Lagrangian–Eulerian type: Lagrangian approach is applied to simulate each particle motion, while Eulerian methods such as a finite difference method are used for the simulation of gas flow.

Vortex in Cell (VIC) method with redistribution of vortex element is one of the vortex methods to simulate incompressible flows [2]. It discretizes the vorticity field with vortex elements and computes the time evolution of the flow by tracing the convection of each vortex element through the Lagrangian approach. The Lagrangian calculation markedly reduces the numerical diffusion as well as ensures the higher numerical stability. Therefore, the VIC method is expected to be usefully employed for the direct numerical simulation (DNS) of turbulent flows and various results have been reported [3–5]. Cottet and Poncet [3] applied the VIC method for the wake simulation of a circular cylinder, and captured the streamwise vortices occurring behind the cylinder. Cocle et al. [4] analyzed the behavior of two vortex system near a solid wall, and made clear the interaction between two counter-rotating vortices and the eddies induced in the vicinity of the wall. Chatelain et al. [5] simulated trailing edge vortices, and visualized the unsteady phenomena caused by disturbances. These studies are concerned with the time-developing free shear flows. But the VIC method has not been applied to the turbulent flows bounded by solid walls, which are closely related with the turbulent friction and the heat transfer. Thus, the authors [6] performed the DNS for a turbulent channel flow, which is a representative example for the wall turbulent flows. When applying the classic VIC method, the oscillation of the flow increased with the progress of the computation, and eventually the computation collapsed. This was caused by the fact that the consistency among the discretized equations is not ensured. This was also because the solenoidal condition for the vorticity is not fully satisfied. To overcome such problems of the existing VIC method, the authors [6] proposed two improvements for VIC method to heighten the numerical accuracy and efficiency: A discretization method employing a staggered grid was presented to ensure the consistency among the discretized equations as well as to prevent the numerical oscillation of the solution, and a correction method for vorticity was also proposed to compute the vorticity field satisfying the solenoidal condition. The improved VIC simulation for the turbulent channel flow highlighted that the time evolution of the flow is fully performed and that the statistically steady turbulent flow is favorably obtained. It also demonstrated that the organized flow structures, such as the streaks and the streamwise vortices appearing in the near wall region, are successfully captured and that the turbulence statistics, such as the mean velocity and the Reynolds stress, agree well with the existing DNS results. The author [7] applied the method to simulate the motion of small air bubbles in water and the behavior of a vortex ring launched toward the bubbles. The simulation highlighted that the bubbles are entrained into the vortex ring and that the entrained bubbles are transported by the convection of the vortex ring.

The objective of this study is to propose a simulation method, which is based on the improved VIC method, for incompressible gas flow laden with small solid particles. It computes the time evolution of the flow by calculating the behavior of the vortex element as well as the particle motion with the Lagrangian approach. Though Walther and Koumoutsakos [8] proposed a VIC method for particle-laden gas flow, the present method based on the improved VIC method is

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http://dx.doi.org/10.1016/j.powtec.2012.10.051

expected to have higher numerical accuracy. It should also be noted that the present method is considered to be usefully applied to the DNS for particle-laden turbulent flow. Because the applicability and validity of the method were demonstrated through the DNS for a single-phase turbulent channel flow [6]. This study also applies the proposed method to simulate a free fall of small spherical solid particles in air so as to demonstrate the validity of the method.

2. Basic equations and numerical method

2.1. Governing equations for gas and particle

When the gas-phase is incompressible, the mass and momentum conservation equations are expressed as:

$$\nabla \cdot u_g = 0 \tag{1}$$

$$\frac{\partial u_g}{\partial t} + \left(u_g \cdot \nabla\right) u_g = -\frac{1}{\rho_g}\nabla p + \nu \nabla^2 u_g - \frac{1}{\rho_g} F_D \tag{2}$$

where F_D is the force exerted by the particle acting on the gas-phase per unit volume.

Taking the curl of Eq. (2) and substituting Eq. (1) into the resultant equation, the vorticity equation for the gas is derived:

$$\frac{\partial \omega}{\partial t} + \nabla \cdot \left(\omega u_g\right) = \nabla \cdot \left(u_g \omega\right) + \nu \nabla^2 \omega - \frac{1}{\rho_g} \nabla \times F_D \tag{3}$$

where ω is the vorticity.

It is postulated that the dominant forces on the particles are the drag and gravitational forces. It is also assumed that the virtual mass force, the Basset force, and the pressure gradient force are negligible. The equation of motion for a particle is written as:

$$\frac{\mathrm{d}u_p}{\mathrm{d}t} = \frac{f}{\tau_p}\left(u_g - u_p\right) + \left(1 - \frac{\rho_g}{\rho_p}\right) g \tag{4}$$

where τ_p is the particle response time, $\rho_p d^2/(18\rho_g \nu)$. f is the drag factor defined by the following equation using the drag coefficient C_D and the particle Reynolds number Re_p $(=d|u_g - u_p|/\nu)$:

$$f = C_D \mathrm{Re}_p / 24. \tag{5}$$

According to Schiller and Naumann [9], f at $\mathrm{Re}_p \leq 800$ is given as:

$$f = 1 + 0.15\mathrm{Re}^{0.687}. \tag{6}$$

Considering Eq. (4), the drag force acting on a particle, F_p, is given as:

$$F_p = \frac{\pi}{6} d^3 \rho_p \frac{f}{\tau_p}\left(u_g - u_p\right). \tag{7}$$

The number of particles per unit volume, n_p, is written by the following equation:

$$n_p = \alpha_p / \frac{\pi}{6} d^3 \tag{8}$$

where α_p is the particle volume fraction.

Since the force F_D in Eqs. (2) and (3) is expressed as $n_p F_p$, the following relation is derived from Eqs. (7) and (8).

$$F_D = \alpha_p \rho_p \frac{f}{\tau_p}\left(u_g - u_p\right) \tag{9}$$

2.2. Orthogonal decomposition of velocity and discretization of vorticity

According to the Helmholtz theorem, the gas velocity u_g is represented as the summation of the gradient of a scalar potential ϕ and the curl of a vector potential ψ: the gas velocity u_g is expressed as:

$$u_g = \nabla \phi + \nabla \times \psi. \tag{10}$$

The velocity calculated from Eq. (10) remains unaltered even when any gradient of scalar potential function is added to ψ. To remove this arbitrariness, a solenoidal condition is imposed on ψ:

$$\nabla \cdot \psi = 0 \tag{11}$$

Taking the curl of Eq. (10) and substituting Eq. (11) into the resultant equation, the vector Poisson equation for ψ is derived:

$$\nabla^2 \psi = -\omega. \tag{12}$$

When substituting Eq. (10) into Eq. (1) and rewriting the resultant equation by using the relation $\nabla \cdot (\nabla \times \psi) = 0$, the Laplace equation for ϕ is obtained:

$$\nabla^2 \phi = 0. \tag{13}$$

When ψ and ϕ are calculated by solving Eqs. (12) and (13) respectively, the velocity u_g is computed from Eq. (10). The vorticity ω in Eq. (12) is estimated from Eq. (3). The Vortex in Cell (VIC) method discretizes the vorticity field with vortex elements to calculate the distribution of ω by tracing the convection of each vortex element.

It is postulated that the position vector and vorticity for the vortex element v are $x_v = (x_v, y_v, z_v)$ and ω_v respectively. The Lagrangian form of the vorticity equation, Eq. (3), is written as:

$$\frac{\mathrm{d}x_v}{\mathrm{d}t} = u(x_v) \tag{14}$$

$$\frac{\mathrm{d}\omega_v}{\mathrm{d}t} = \nabla \cdot (u(x_v)\omega(x_v)) + \nu \nabla^2 \omega(x_v) - \frac{1}{\rho_g} \nabla \times F_D. \tag{15}$$

When the position and vorticity of vortex element are known at $t = t$, the values at $t = t + \Delta t$ are computed from Eqs. (14) and (15). In the VIC method, the flow field is divided into computational grid cells to define ψ, ϕ and ω on the grids. If ω is defined at a position $x_k = (x_k, y_k, z_k)$, the vorticity ω is given to x_k, or a vortex element with ω is redistributed onto x_k

$$\omega(x_k) = \sum_{v}^{N_v} \omega_v W\left(\frac{x_k - x_v}{\Delta x}\right) W\left(\frac{y_k - y_v}{\Delta y}\right) W\left(\frac{z_k - z_v}{\Delta z}\right) \tag{16}$$

where N_v is the number of vortex elements, and Δx, Δy and Δz are the grid widths. For the redistribution function W, the following equation is employed [10]:

$$W(\varepsilon) = \begin{cases} 1 - 2.5\varepsilon^2 + 1.5|\varepsilon|^3 & |\varepsilon| < 1 \\ 0.5(2 - |\varepsilon|)^2 (1 - |\varepsilon|) & 1 \leq |\varepsilon| \leq 2 \\ 0 & |\varepsilon| > 2 \end{cases}. \tag{17}$$

2.3. Discretization by using staggered grid

For incompressible flow simulations, the MAC method and the SMAC method solve the Poisson equation for pressure, which is derived from the equation for pressure gradient and the continuity equation. They employ a staggered grid to ensure the consistency between the discretized equations as well as to prevent the numerical oscillation of the solution.