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An investigation of the comparative behaviour of alternative contact force models during inelastic collisions

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ABSTRACT

Rebound kinematics are compared for both viscous dissipation and plastic dissipation models for an inelastic sphere obliquely impacting a target wall for a range of normal coefficients of restitution. The models are quite consistent for high normal coefficients of restitution but significant differences are noted between the models as the normal coefficient of restitution reduces. The reasons for the differences, in terms of the rebound tangential surface velocities, are explained for both plastic and viscous dissipation models. A new 'partially latching spring' model is proposed that provides realistic predictions of the contact force magnitude and contact duration. Finally, a general condition for sliding to occur throughout inelastic impacts is identified for any value of the normal coefficient of restitution.

1. Introduction

Flows of granular material are widespread in both nature and industry and the numerical modelling of particle flows is therefore of significant scientific and industrial interest. Particle flows involve discrete particles that interact with each other and these particle scale interactions control the micromechanical behaviour that leads to emergent properties observed at the macroscale. Consequently, in order to understand the connection between the microscale and macroscale behaviour there has, over the last three decades, been an ever increasing trend to try to model particulate material as a discontinuum rather than using the more traditional continuum modelling approach. In this context, the most popular numerical modelling technique is the Discrete Element Method (DEM), originally developed by Cundall and Strack [1] for quasi-static deformation of compact particle systems. It is now used for a wide variety of problems that may involve collisional systems (rapid granular flow), particles with enduring contacts (quasi-static deformation of compact particle systems) or both (the intermediate flow regime), see papers published in [2–4].

In DEM simulations, the translational and rotational motions of all individual particles are continuously tracked using Newton's second law of motion in which the accelerations are functions of the forces at the inter-particle contacts. The actual forces at the contacts between contiguous particles depend on the particle–particle interaction rule used, i.e. the models used calculate the normal and tangential forces at a contact. Currently, there are a number of different contact force models that have been implemented for DEM simulations and the contact interaction laws invariably incorporate some form of energy dissipation, which can be either viscous or plastic. However, it is currently unclear what significance the choice of contact force model has on the resulting flow or its properties.

The work reported in this paper is part of an ongoing project designed to rigorously assess the significance of the detailed formulation of particle–particle interactions. Clearly the significance may depend on whether the problem involves collisions, enduring contacts or both. It is therefore extremely difficult to truly validate DEM simulations. Consequently, as a start, we examine the apparently simplest of problems, which is the oblique impact of a sphere with a target wall. Thornton et al. [5] compared the rebound characteristics obtained using various contact force models for oblique elastic impacts. In this paper, we examine inelastic oblique impacts using viscous dissipation models and plastic dissipation models. The aim of the paper is to show how the rebound characteristics depend on the choice of model used and to provide some explanations for the differences found.

Previous comparisons of the effect of different dissipative particle interaction models were reported by Schäfer et al. [6] and more recently by Kruggel-Emden et al. [7]. Schäfer et al. [6] used a linear spring-dashpot for the normal force model with a normal coefficient of restitution $e_n = 0.87$. They examined various tangential force models and compared the results with the experimental data of Foerster et al. [8]. It was demonstrated that viscous tangential force models, commonly used by physicists at that time [9–12] fail to predict the negative rebound tangential surface velocities observed in

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experiments since they do not take account of tangential elasticity. An exception was the viscous tangential force model of Brilliantov et al. [13] but this model significantly over-predicted the positive rebound tangential surface velocities when sliding occurred throughout the impact. Better agreement with the experimental data was obtained, for all impact angles, using the elastic tangential force model of Walton and Braun [14] and a linear tangential spring model that was incorrectly attributed to Cundall and Strack [1], see below.

Kruggel-Emden et al. [7] compared the results of several tangential force models [1,13–18] with experimental data [8,18–22]. They showed that, when sliding occurred throughout the impact, the viscous tangential force model of Brilliantov et al. [13] not only predicted significantly excessive positive rebound tangential surface velocities, as found by Schäfer et al. [6], but also significantly under-predicted the angular velocity imparted to the impacting sphere. Consequently, viscous tangential force models will not be considered in this paper.

Kruggel-Emden et al. [7] used various dissipative normal force models to fit the normal coefficients of restitution observed in the different experiments and examined the ability of different elastic tangential force models to match the experimental data. They concluded that the 'best' models were the 'constrained' linear model, the 'constrained' model of Di Renzo and Di Maio [18] and the tangential force model of Walton and Braun [14]. However, it has been demonstrated by Thornton et al. [5] that the so-called 'constrained' models of Di Renzo and Di Maio [17,18] are mathematically incorrect interpretations of the theory of Mindlin [23].

In this paper we consider normal and tangential contact force models that have not previously been compared and, for each model, we examine the effect of a much wider range of values for the normal coefficient of restitution than previously considered.

2. Contact force models

The most common contact force model, used for both normal and tangential interactions, is the linear spring–dashpot model introduced by Walton [24]. (Many researchers mistakenly attribute the linear spring–dashpot model to Cundall and Strack [1]. Although dashpots were used by Cundall and Strack [1] they did not contribute to the contact forces. The contact forces were simply the forces in the springs. However, the dashpot forces were added to the spring forces to provide the contribution to the particle out-of-balance force to be used to calculate particle accelerations. The original purpose of the dashpots was to suppress 'rattling' at contacts during 2D quasi-static simulations.) The linear spring–dashpot models are widely used due to their ease of implementation in numerical codes and their robustness. Non-linear variations of these models have also been proposed and implemented e.g. Tsuji et al. [15] and Zhou et al. [25]. These dashpot models dissipate energy through viscous means. Other normal interaction models, Walton and Braun [14], Stronge [26] and Thornton [27], dissipate energy plastically via the use of different loading and unloading spring stiffnesses. Firstly, we summarise the theoretical basis for each of the different models to be examined.

2.1. Models based on contact mechanics

For elastoplastic spheres the theoretical model of Thornton [27] is used to provide the normal force–displacement relationship, see [28,29] for further details.

The initial normal interaction is elastic with the normal force and the radius of the contact area given by

$$F_n = \frac{4}{3} E^* R^{1/2} \alpha^{3/2} \tag{1}$$

$$a = \sqrt{R\alpha} \tag{2}$$

where α is the relative displacement (approach), R is the sphere radius and

$$E^* = \frac{E}{2(1-\nu^2)}$$

where E is Young's modulus and ν is Poisson's ratio.

Thornton [27] suggested that the normal interaction becomes plastic when a 'limiting contact pressure' p_y is reached at the centre of the contact area, as shown in Fig. 1a. This idea originates from Hardy et al. [30] who reported results of a finite element analysis of a rigid sphere indenting an elastoplastic half-space. They showed that the Hertzian pressure distribution is valid until the pressure at the centre of the contact area is equal to 1.6 times the yield stress of the material, at which point yield occurs below the centre of the contact area. Further compression results in a spreading of the plastic deformation zone below the surface and a slight modification of the shape of the contact pressure distribution as the maximum contact pressure increases further. When the pressure at the centre of the contact area reaches about 2.4 times the yield stress, the plastic deformation zone in the substrate reaches the contact surface at the perimeter of the contact area. Beyond this point, further compression results in a significant change in the form of the pressure distribution. Over an increasing central portion of the contact area the contact pressure becomes almost constant with only a slight increase in the pressure at the centre of the contact area.

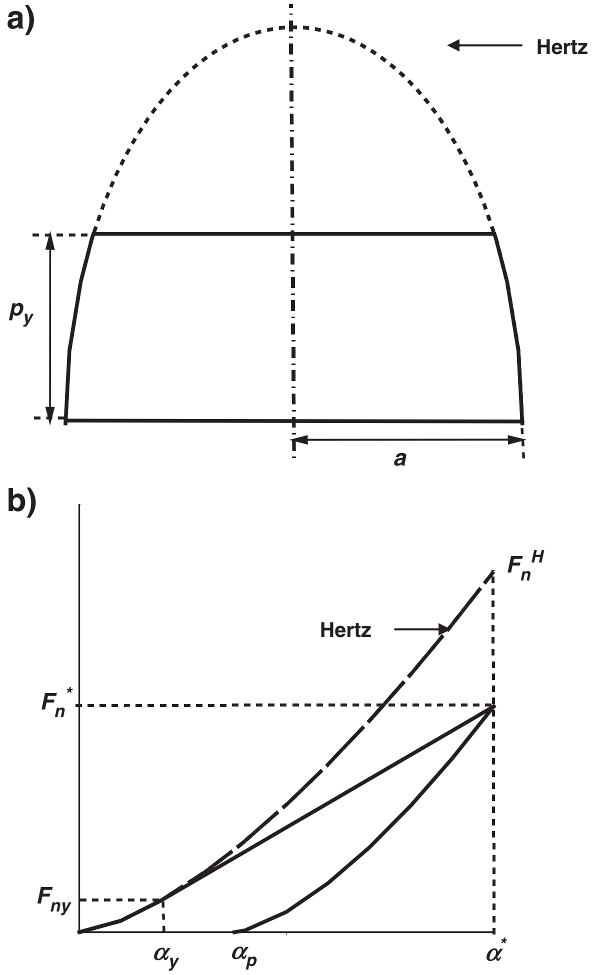

Fig. 1. Normal interaction details for the theoretical model of Thornton [27] (a) normal pressure distribution (b) force–displacement curves.