Powder Technology 221 (2012) 123–136

Contents lists available at SciVerse ScienceDirect

Powder Technology

journal homepage: www.elsevier.com/locate/powtec

Lode dependency in the cold die powder compaction process

S.A. Rolland [a,*], P. Mosbah [b], D.T. Gethin [a], R.W. Lewis [a]

[a] College of Engineering, Swansea University, Swansea, UK
[b] ISEN, Lille, France

ARTICLE INFO

Article history:
Received 11 November 2011
Accepted 17 December 2011
Available online 26 December 2011

Keywords:
Constitutive model
Plasticity
Powder compaction
Lode dependency

ABSTRACT

The powder forming industry looks to produce parts of increasing geometrical complexity as it is seen as a very efficient production process. This offers new challenges as three-dimensional states of stress are induced. In particular, granular and porous materials respond very differently to tensile and compressive stresses. Since experiments conducted in the 1990s, little exploration of the Lode dependency of powders was carried out. The present work investigates the effect of Lode dependency through numerical simulation, aiming to establish whether it affects the outcome of a compaction cycle and whether further experimental study of the phenomenon may be justified. To this effect, a Lode dependent model was developed and implemented in a finite element code, then two case studies were carried out. The results show that there is little impact on the density contours within the components and the stress levels during the compaction. As the parts are ejected from the die, surface stress levels are affected and this is of great interest when studying the onset of defects in powder compacts.

1. Introduction

The cold die powder compaction manufacturing process is a popular manufacturing route for small and relatively geometrically simple components. This is due to its high material usage as the parts are produced to near finished dimensions from the powder. The manufacturing process is a very challenging one which can be investigated using several analytic disciplines of engineering: fluid [1] and solid mechanics [2] with discrete [3] or continuous approaches [2], [4]. Other empirical techniques can be also considered [5]. The present document concerns itself with the forming phase of the process, starting when the die has been filled with powder and ending when the compact is fully ejected. This process is well documented experimentally and analytically [6,7]. The analysis technique chosen is based on the finite element analysis of axisymmetrical compacts. The geometrical choice is due to industrial relevance and the simplifications it allows in the modelling process. The choice of the technique is due to the broad range of differential equation based problems that it can solve. A solid mechanics formulation based on large strain and a plasticity formulation is used to model the irrecoverable deformations. This choice is supported by numerous publications demonstrating the ability of plasticity models to simulate powder behaviour. Experimental and numerical investigations are often conducted closely in fields of industrial relevance and many publications show that this applies to simulations of powder compaction and models can now produce reliable density contours [8]. The results are however less reliable regarding the prediction of tools forces, where relatively large error margins are still encountered. An experimental study conducted by Mosbah et al. showed that ejection forces and residual stresses are usually over-predicted by models than actually encountered in experiments [9]. The authors concluded that the effects of Lode dependency were disregarded and may have had an important role, this result can be intuitively understood as granular materials are not expected to respond in the same fashion if they are under tensile or compressive stress. This conclusion was later supported by the work of Shima, which demonstrated experimentally that the powder behaviour is influenced by the third deviatoric stress invariant, and is therefore Lode dependent [10,11]. Although the latter work supports the hypothesis formulated by Mosbah et al., it is not sufficient to confirm it. Due to the difficulties of measuring residual stresses in green compacts, by nature very fragile, experimentalists have since given very little attention to this topic. The present work aims to investigate the extent to which the Lode angle influences the behaviour of the material during the forming phase of the process. To do so, a Lode dependent numerical model was developed for powders and implemented in a finite element code. Two test cases were selected and their compaction cycle simulated to quantify the impact of Lode dependency, the first one is a simple cylinder and the second one a stepped component. Particular attention was given to the internal stresses and tool forces. The work does not aim to give an exact simulation of Lode dependency at the current stage of research as there is too little experimental work to support a particular Lode dependent model. Rather, the present work seeks to establish whether there is cause to explore further

* Corresponding author. Tel.: +44 1792606871.
E-mail address: s.rolland@swansea.ac.uk (S.A. Rolland).

0032-5910/$ – see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.powtec.2011.12.044

Lode dependency in powder compacts, and if so, the context in which it matters.

2. Basis of elasto-plasticity

Elasto-plastic models are used for the simulations of the powder compaction process although the mechanisms are not strictly elasto-plastic at all times. This approach has been taken historically because it is a convenient way to represent non-recoverable strains incurred under stress. Non linear models, with hardening rules, are particularly suited to simulate the densification process. The mathematical basis of elasto-plasticity is laid out in numerous publications. The reader is referred to Crisfield, for example, as a source of more detailed information [12]. In powder compaction, yield models are particularly challenging due to their dependence on density or potentially other state variables such as work, which governs the material's hardening. Classical elasto-plasticity yield models are based on an elasticity domain, the decomposition of strains and a yield criterion. It is assumed that the strain increments can be decomposed in their elastic and plastic fractions, $\Delta\epsilon^e$ and $\Delta\epsilon^p$:

$$\Delta\epsilon = \Delta\epsilon^e + \Delta\epsilon^p \tag{1}$$

in which superscript ^{e} refers to fraction of the strain increment and the superscript ^{p} refers to plastic strain fraction. The strain decomposition can be substituted into Hooke's law of elasticity:

$$\Delta\sigma = C\Delta\epsilon^e = C\left(\Delta\epsilon - \Delta\epsilon^p\right) \tag{2}$$

in which σ is the stress vector and C is the matrix of elastic coefficients. The set of admissible stresses is governed by a yield function f. Plastic flow occurs if $f=0$. The function f cannot take a positive value as it bounds the admissible stresses. If $f<0$, plastic loading does not occur and there is no plastic flow. In this case the behaviour is elastic and the stresses can be updated by:

$$\sigma_k = \sigma_{k-1} + \Delta\sigma = \sigma_{k-1} + C\Delta\epsilon = \sigma_{k-1} + C\left(\Delta\epsilon - \Delta\epsilon^p\right).$$

When plastic loading occurs, it is governed by a flow rule:

$$\Delta\epsilon^p = \lambda\frac{\partial g}{\partial\sigma}$$

in which g is the plastic potential function and λ the plastic multiplier. In this work, the plastic potential and the yield surface are considered associated, so $f=g$. Additionally, the model is rate independent, so the equivalence between the rate equations and the incremental form can be exploited. From the flow rule and Hooke's law, the stress–strain relationship can be written in the case of plastic loading:

$$\Delta\sigma = C\left(\Delta\epsilon - \lambda\frac{\partial f}{\partial\sigma}\right). \tag{3}$$

This expression leads to the formulation of the elastic prediction for the incremented stresses and to the basis for a return to the yield surface when the elastically incremented stresses are not in the elastic domain:

$$\sigma = \sigma_0 + C\left(\Delta\epsilon - \lambda\frac{\partial f}{\partial\sigma}\right) \tag{4}$$

in which the elastically predicted stress is $\sigma^e = \sigma_0 + C\Delta\epsilon$. The return to the yield surface is carried out through an iterative process. The plastic multiplier must be positive and satisfy the yield function. The equations involved can be highly non linear and the success of the return to the yield surface is dependent on the yield function, which must be formulated to ensure the unicity of the solution.

3. The modified CamClay model for powders

There are a number of yield surface forms that may be adopted. Common forms include an ellipse, e.g. modified CamClay by Lewis and Schrefler or Mosbah et al. [13,9], and two-surface models such as the Drucker–Prager model enclosed by a shear failure envelope and a compaction cap [14]. In the modified CamClay model available in the literature, the yield surface can be expressed as a function of the pressure, P, and equivalent stress, Q:

$$f = \frac{(P-P_1)^2}{P_0^2} + \frac{Q^2}{Q_0^2} - 1$$

P_0, P_1 and Q_0 are coefficients governing the material hardening properties. For convenience of use when introducing the third deviatoric stress invariant, this can be rewritten as a function of the first stress invariant and the second deviatoric stress invariant:

$$f = 27J_2 + M(I_1 - I_c)(I_1 - I_0) \tag{5}$$

in which $I_1 = \sigma_{ii} = -3P$ is the first stress invariant, $J_2 = \frac{\bar{\sigma}_{ij}\bar{\sigma}_{ij}}{2} = \frac{Q^2}{3}$ is second deviatoric stress invariant with the deviatoric stress matrix $\bar{\sigma} = \sigma - \frac{I_1}{3}I$. The hardening coefficients are: $I_0 = -3(P_0 + P_1)$, $I_c = -3(P_1 - P_0)$, and $M = \frac{Q_0^2}{P_0^2}$. They are functions of the density which is used as a hardening variable. The model is associated, thus the derivative of the plastic potential for the flow rule is:

$$\frac{\partial f}{\partial\sigma_{ij}} = 27\bar{\sigma}_{ij} + M(2I_1 - I_0 - I_c)\delta_{ij}$$

where δ_{ij} is the Kronecker delta function. The stress increments in Eq. (3) can be written in a tensor form as:

$$\Delta\sigma_{ij} = C_{ijkl}\left(\Delta\epsilon_{kl} - \lambda\frac{\partial f}{\partial\sigma_{kl}}\right). \tag{6}$$

The development of the term $C_{ijkl}\frac{\partial f}{\partial\sigma_{kl}}$ from the equation above gives:

$$C_{ijkl}\frac{\partial f}{\partial\sigma_{kl}} = \frac{E}{1+\nu}\frac{\partial f}{\partial\sigma_{ij}} + \frac{\nu E}{(1-2\nu)(1+\nu)}tr\left(\frac{\partial f}{\partial\sigma_{mn}}\right)\delta_{ij}.$$

With $tr\left(\frac{\partial f}{\partial\sigma_{mn}}\right) = 3M(2I_1 - I_0 - I_c)$, this leads to:

$$C_{ijkl}\frac{\partial f}{\partial\sigma_{kl}} = \frac{E}{1+\nu}\left(27\bar{\sigma}_{ij} + M(2I_1 - I_0 - I_c)\delta_{ij}\right) + \frac{3\nu ME(2I_1 - I_0 - I_c)}{(1-2\nu)(1+\nu)}\delta_{ij}$$
$$C_{ijkl}\frac{\partial f}{\partial\sigma_{kl}} = \frac{27E}{1+\nu}\bar{\sigma}_{ij} + \frac{ME}{1-2\nu}(2I_1 - I_0 - I_c)\delta_{ij}. \tag{7}$$

In a fully implicit integration, the stress is considered at the end of the step and substituting Eqs. (6) and (7) into Eq. (4) gives:

$$\sigma_{ij} = \sigma_{ij}^e - \lambda\left[\frac{27E}{1+\nu}\bar{\sigma}_{ij} + \frac{ME}{1-2\nu}(2I_1 - I_0 - I_c)\right]. \tag{8}$$

Eq. (8) can be split into the hydrostatic and deviatoric contributions. The hydrostatic part is given by:

$$I_1 = I_1^e - \lambda\frac{3ME}{1-2\nu}(2I_1 - I_0 - I_c).$$

From this, the expression for λ is:

$$\lambda = \frac{(I_1^e - I_1)(1-2\nu)}{3ME(2I_1 - I_0 - I_c)}. \tag{9}$$