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Dynamics in the boundary layer of a flat particle

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A B S T R A C T

The paper presents a theoretical study of the source of the spinning movement of the solid particles flowing in a moving fluid and the influence of the resulting Magnus force on the particles' trajectories along the stream lines, based on interactions occurring in the boundary layers. The subject is important for technological applications, like aerodynamic separation process of a mixture of solid particles. First, it is shown that the boundary layer equations generate local soliton-type, kink-type and soliton-kink-type nonlinear solutions for the velocity field. Using Prandtl's equations for boundary layer, nonlinear solutions of the velocity field are obtained. It was found that through the interaction on the boundary layers, the transition from the movement on continuous and differentiable curves (stream lines) to the movement on continuous and non-differentiable curves (fractal curves) occurs. This last characteristic can be used in the separation process of the solid components from a heterogeneous mixture.

1. Introduction

Flows including solid particles are often observed in industry and nature. Whatever the source, a mixture of solid particles is a heterogeneous mixture with components having different properties. Knowledge of the exact components helps the correct choice of the separation processes and techniques. When solid mixture components are different by their behavior in the air flow, the separation can be achieved through aerodynamic properties. The knowledge of these properties is important because they represent the base for the proper choice and adjustment of air flow rate, for the calculation and design of pneumatic systems used for sorting, cleaning or transport of the mixture.

Aerodynamic separation process of a mixture of solid particles has applications both in industry [1–3] and agriculture [4–7]. Recently, the researches in the field placed particular emphasis on environmental protection, one of the studied issues in this direction being the optimization of air depollution installations [8]. Many studies investigated the influence of dimension, form and density of solid particles on speed flow [9–15].

Solid particles may change the flow characteristics and accordingly affect the transfer of energy [16–19]. Moreover, the turbulence mechanism of particle-laden flows [20,21] is not yet well understood [18,22].

The particle rotation is an important factor in the flow involving fluids and solid particles because the lift force influences the particle distribution [23–25]. Usually, flow processes don't take into account the impact of the dynamics in the boundary layer. Thus, this paper presents a theoretical study of the source of the spinning movement of the solid particles flowing in a moving fluid and the influence of the resulting Magnus force on the particles' trajectories along the stream lines, based on interactions occurring in the boundary layers. For this, the motion of a flat solid particle in a real fluid stream with constant velocity was considered. It was assumed that the flow from the boundary layer around the solid particle is bi-dimensional $\boldsymbol{V}(x,y)=u(x,y)\boldsymbol{i}+w(x,y)\boldsymbol{j}$. The fluid rotation movement from the boundary layer is transmitted, through the friction effect, to the particle. Thus, around the particle appears the circulation of the velocity Γ (Magnus effect) and, accordingly, a lift force. As a result of rotational speed induced on particle, the particle acquires additional kinetic energy that allows "jumps" from its own stream line to another. The knowledge of the particles' movement on continuous and non-differentiable curves (fractal curves) can be used to explain the separation mechanism of the solid components from a heterogeneous mixture.

The study is structured as follows: in Section 2 considerations on generation and dynamics analysis in the boundary layer are made;

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Prandtl's equations for boundary layer and nonlinear solutions of the velocity field are obtained in Section 3; in Section 4 the force field induced by Magnus effect is given.

2. Considerations on generation and dynamics analysis in the boundary layer

Let us consider a solid particle with negligible weight having the order of magnitude of the surface S of 10^{-6} $[m^2]$ and a real fluid with the viscosity coefficient $\nu = 14 \cdot 10^{-6}$ $[m^2/s]$. The velocity of the fluid current $V_0[m/s]$ is constant. As a consequence of the bi-dimensional and permanent character of the movement, in any point of the fluid mass the local velocity can be written as:

$$\mathbf{V}(x,y) = u(x,y)\mathbf{i} + w(x,y)\mathbf{j}.$$

If the Reynolds number associated to the flow is big enough, but still smaller than the critical value, then the movement in its ensemble has a laminar character. As a consequence, the dynamic equilibrium equations which describe the movement are [18,22]:

$$\begin{aligned} u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial y} + \frac{1}{\rho}\frac{dp}{dx} &= \nu\frac{\partial^2 u}{\partial x^2} + \nu\frac{\partial^2 u}{\partial y^2} \\ u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial y} + \frac{1}{\rho}\frac{dp}{dy} &= \nu\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}. \end{aligned} \tag{1a, b}$$

The continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial y} = 0 \tag{2}$$

is added to the equation system (Eq. (1 a, b)). In the system of Eq. (1 a, b) the unknown variables are the scalar components $u(x,y)$ and $w(x,y)$ of the local velocity $\boldsymbol{V}(x,y)$.

In the above mentioned conditions, the real fluid flowing around the solid particle determines the formation of the limit layer around it. In the limit layer the particle movement takes place naturally along a current line, which is a differentiable curve given by the equation:

$$\frac{dx}{u(x,y)} = \frac{dy}{w(x,y)} = const. \tag{3}$$

If the energy transfer from the moving real fluid to the particle is high enough, this can jump from a current line to another. In this case, the movement takes place on non-differentiable curves, i.e. on fractal curves. Let $\delta(x)$ be the local thickness of the limit layer formed on one side of the particle. The interaction between the limit layers of two particles which approach each other at a distance $\delta(x) < \varepsilon < 2\delta(x)$ can be analyzed.

In fluids in motion, the particles which build the base of the continuous medium are parts of the fluid. Their dimensions are considered in such a way that the average characteristics of the motion (in the volume V) satisfy certain smoothing conditions (the functions which define them and their derivatives are continuous to a certain order). In the case of sub-particles, these measures present certain jumps and sudden variations in connection with the sensitive fluctuant number of molecules from sub-particles. As a consequence, the average characteristics do not represent either information at molecular level, or information at the continuous medium level. In deducing Navier–Stokes equations for the continuous fluid medium, the dimension of the particles and the distance between them does not intervene either way, due to the fact that these aspects were not taken into account when establishing the bijective correspondence between the particle set and the Euclidian space set $\Re^3$. Experimentally though, it is possible to determine the particles' dimension corresponding to the continuous medium model; this dimension is called “scale of the fluid movement”.

A particular continuous fluid medium model is that of the ideal fluid for which, instead of the Navier–Stokes equations, Euler's equations are used. According to this model, the fluid particle must be characterized by:

$$\int_{\mathcal{V}} \nabla \times \mathbf{V}(x,y) d\mathcal{V} = 0.$$

There is a potential model in any fluid movement, but not at any dimension scale.

The potential models of permanent incompressible movements around obstacles represent solutions of the boundary problem

$$\alpha \mathbf{V}_c + \beta \left[\frac{\partial \mathbf{V}_c}{\partial n}\right] = \mathbf{\Phi}_c; \mathbf{V}_0 = \mathbf{\Phi}_0 \tag{4}$$

for the Laplace equation:

$$\Delta \mathbf{V} = 0,$$

where $\boldsymbol{V} = \boldsymbol{V}(x,y,z)$ is the velocity, $(x,y,z) \in D$ is the movement domain, C is the boundary of D and has the external normal $\boldsymbol{H}$. The index c shows that the values are taken on the contour C; the index 0 shows that the values are taken at long distances while α and β are constants. When the functions $\boldsymbol{V}_C$; $\boldsymbol{V}_0$; $\boldsymbol{\Phi}_C$; $\boldsymbol{\Phi}_0$; and C are regular functions, the elliptic Eq. (4) has a unique solution determined by those measures. In the case when these equations of type (4) present singularities, solutions can be built in the regularity domain (coherence) by isolating neighborhoods of the singularities. The solution in one point describes the behavior of the particle centered in that point and having the dimension (order of magnitude) $\delta < 10^{-2} L$ where L is the characteristic dimension of the body. This solution is physically possible if and only if [26,27]:

i) in any domain, no matter how small, where:

$$\nabla \times \mathbf{V} = 0$$

is punctual;

ii) in domains of volume V where formula

$$\int_{\mathcal{V}} \nabla \times \mathbf{V} d\mathcal{V} = 0$$

is satisfied;

iii) in the domain with maximum dimension for which $\boldsymbol{V}_0 = \text{const}$.

If one gets lower than $V_{inferior}$, then we have:

$$\int_{\mathcal{V}} \nabla \times \mathbf{V} d\mathcal{V} \neq 0$$

for any $V \subset V_{inferior}$. This fact leads us to the conclusion that the dimensional scale is given by $V_{inferior}$. As a consequence, as long as we are not interested in the local behavior, the global characterization through the condition

$$\int_{\mathcal{V}} \nabla \times \mathbf{V} d\mathcal{V} = 0$$

is sufficient and the potential solution is acceptable. Its physical significance ceases when the dimension d of the particles is no longer the same in the entire field of motion.

The potential solution of the movement corresponds to a minimum of kinetic energy, fact that explains the tendency of the fluids to correspond to this solution at global scale. By “optimizing the energy at global scale” we understand that the perturbations due to the presence of the