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Radial porosity in packed beds of spheres

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ABSTRACT

A functional approach has been developed to investigate the radial porosity of mono-sized spheres in cylinders. Analytical and semi-analytical equations have been developed to calculate the local radial porosity and the radial porosity distribution, respectively, within a cylindrical packing structure. The analytical equations are based upon fundamental principles and are simple, straightforward and provide highly accurate results for the radial porosity with minimal computational prerequisites. The analytical equations have been developed for the fixed packing of identical spheres in cylindrical containers with $D/d \geq 2.0$. The predicted results for the local radial porosity and the radial porosity distributions are benchmarked with an existing analytical equation and available experimental data, respectively, for mono-sized spheres in cylindrical containers.

1. Introduction

The axially-averaged radial porosity variation is present in confined packed systems which are comprised of non-spherical or spherical particles. The radial porosity variation is a characteristic structural feature of confined fixed packed beds which occurs because of the influence of the container walls. It is universally recognized that the wall effects on the local porosity can be a significant constituent in the design and analysis of confined packed systems which include, for example, chromatography columns, chemical and nuclear reactors, and thermal heat exchangers. In particular, the radial porosity variations for mono-sized spheres in cylindrical containers are regularly used and have been investigated using various experimental and systematic methods by Roblee et al. [1]; Benenati and Brosilow [2]; Thadani and Peebles [3]; Ridgway and Tarbuck [4]; Martin [5]; Cohen and Metzner [6]; Goodling et al. [7]; Kufner and Hofmann [8]; Mueller [9]; Govindarao et al. [10]; Sederman et al. [11]; Wang et al. [12]; and Mariani et al. [13]. These experimental packing data provide an indispensable resource for benchmarking the accuracy of any type of predictive packing structural expression for spheres in cylindrical containers.

Packed bed hydrodynamic and heat transport numerical modeling techniques that analyze the bed as a pseudo-homogeneous media and therefore incorporate the local porosity variation generally involve using the axially-averaged radial porosity distribution. The local porosity variation in the radial direction, which is oscillatory and damped in nature, has been modeled completely by several different empirical approaches. Very simplified radial porosity models that characterize the radial porosity variation and use only an exponential-type function are typically similar to the models given by Vortmeyer and Schuster [14] and de Klerk [15]. The exponential-type radial porosity models do not account for the observed oscillations in the radial porosity and consequently are an average porosity as a function of the radial direction. The more representative radial porosity model developed by Martin [5]; Cohen and Metzner [6]; Kufner and Hofmann [8]; de Klerk [15]; and Bey and Eigenberger [16]; involves using a cosine function to characterize the radial oscillations and an exponential function to describe the radial damping. These model types predict the radial porosity very well near the cylindrical wall, but become less accurate as the radial distance from the wall increases. Mueller [9,17] characterized the oscillations with a Bessel function of the first kind of order zero and the radial damping with an exponential function. This model type over predicts the radial porosity near the cylindrical wall but describes the radial porosity accurately as the radial distance increases from the wall. In addition to the purely empirical models, more rigorous analytical approaches have been employed to characterize the radial porosity variation. Govindarao and Froment [18]; Govindarao and Ramrao [19]; and Govindarao et al. [20] developed expressions to describe the radial porosity variation with concentric cylindrical channels of equal thickness and spherical particle centers so that model accuracy increased with larger diameter aspect ratios. Du Toit [21] developed analytical based numerical procedures to evaluate area-based porosity. Mariani et al. [22–24] developed analytical closed expressions for the radial porosity profiles and the distribution of spherical particle centers. The comprehensive expressions of Mariani et al. [22]

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relate the intersection of a sphere and cylindrical surface in terms of natural functions that arise from integral calculus, i.e. elliptic integrals. The connection between the distribution of spherical particle centers and radial properties in random packed beds were also addressed by Mariani et al. [22], and consequently the results have significantly improved the understanding of this relationship. Nevertheless, even though the analytical expressions developed by Mariani et al. [22] are particularly accurate in predicting the radial porosity variation for spherical particles in cylindrical packed beds, the analytical predictive models have not been widely used in the engineering and scientific community in transport numerical modeling as compared to the less accurate and purely empirical radial porosity models.

As a result, there exists a significant disparity in the predictive radial porosity modeling domain between the less accurate purely empirical models and the more accurate complex analytical expressions. What is needed for transport numerical modeling for local phenomena that incorporates and accounts for the radial porosity variation is a model that is simple to use, accurate in the near and far wall regions, and analytical in nature, i.e., that it has been developed based on fundamental principles. It is for this reason, that an analytical based model can significantly improve the understanding of the packed bed structural configuration, which has been shown to be a function of the diameter aspect ratio, D/d, and decrease the errors propagated in the numerical simulations from using imprecise empirical radial porosity models.

The objective of this study is to present a functional approach based on established fundamental geometrical structural principles that has been developed to investigate the axially-averaged radial porosity variation of mono-sized spherical particles in cylindrical packed beds systems. Analytical expressions are presented for the local porosity variation and a semi-analytical expression for the radial porosity distribution. The new analytical expressions are compared with an existing analytical expression from the literature for the local radial porosity and validated with experimental data for the radial porosity distribution. The analytical expressions have been developed for fixed packed beds of identical spheres in cylindrical containers with $D/d \geq 2.0$.

2. Local radial porosity

Generally, the local radial porosity is characterized as a volumetric structural property of the packing system and has a numerical value between 0 and 1. Whether it is determined experimentally or through some analytical or numerical expression, it is represented as the axially-averaged local radial porosity that is obtained in a radial annular cylinder between the radii r and $r+\Delta r$. The local radial porosity, $\varepsilon(r)$, is normally defined as the fraction of the volume of voids to the total volume in the radial annular cylindrical layer. It is normally evaluated as $\varepsilon(r) = V_{void}/V_{total} = (V_{total} - V_{solid})/V_{total} = 1 - V_{solid}/V_{total}$, where V_{solid}, V_{void}, and V_{total} are the solid volume, void volume, and total volume, respectively, in the radial annular cylindrical layer. For a fixed cylindrical packed bed of mono-sized spherical particles of radius, R_s, with a center radial position of, r_s, this specific geometry condition in the x–y plane with an origin, O, is shown in Fig. 1. For this particular situation, the local radial porosity is expressed in terms of the solid volume contributions of spheres with radial centers positions within a particle radius on either side of the radial annular cylindrical layer. This established approach to calculating the local radial porosity has been used for well over 50 years. One of the principle priorities of this study is to first consider the radial annular cylindrical volume between the radii r and $r+\Delta r$ as shown in Fig. 1. For this local radial porosity analysis, let the radial annular cylindrical thickness, Δr, go to zero, $\Delta r = 0$, and as a result, what occurs is the intersection of a radial cylindrical surface of radius, r, from the origin, O, with that of any sphere with a radial center position, r_s, within a particle radius, R_s, on either side of this radial cylindrical surface. This particular geometrical situation is shown in Fig. 2. A somewhat comparable form of sphere–cylinder geometrical surface interaction has been considered and analyzed by Mariani et al. [22] together with geometrical expressions and quantities that were defined and derived in the investigation. For this current analysis, the local radial porosity, $\varepsilon(r)$, is now defined and evaluated in terms of this intersecting sphere–cylinder area as $\varepsilon(r) = A_{void}/A_{total} = (A_{total} - A_{solid})/A_{total} = 1 - A_{solid}/A_{total}$, where A_{solid}, A_{void}, and A_{total} are the solid area (intersecting area), void area (non-intersecting area), and total area (total radial cylindrical surface area), respectively, on the radial cylindrical surface. The local radial porosity, $\varepsilon(r)$, has now been reduced to obtaining expressions for areas instead of volumes and will result in a more uncomplicated and convenient expression for determining the local radial porosity. Hence, the local radial porosity, which is a function of the radial position, for a cylindrical packed bed system of mono-sized spheres is given by:

$$\varepsilon(r) = 1 - \frac{A_{solid}}{A_{total}} = 1 - \sum_{n=1}^{N} \frac{S_n(r)}{S_T(r)}, \quad (1)$$

where N is the number of sphere particles located within a sphere particle radius, R_s, on either side of the radial cylindrical surface at the radial position, r, $S_n(r)$ is the intersecting area of an nth-sphere at the radial location, r, and $S_T(r)$ is the total radial cylindrical surface area at a radial location, r, and is easily found to be $2\pi rH$ where H is the height of the radial cylindrical surface area.

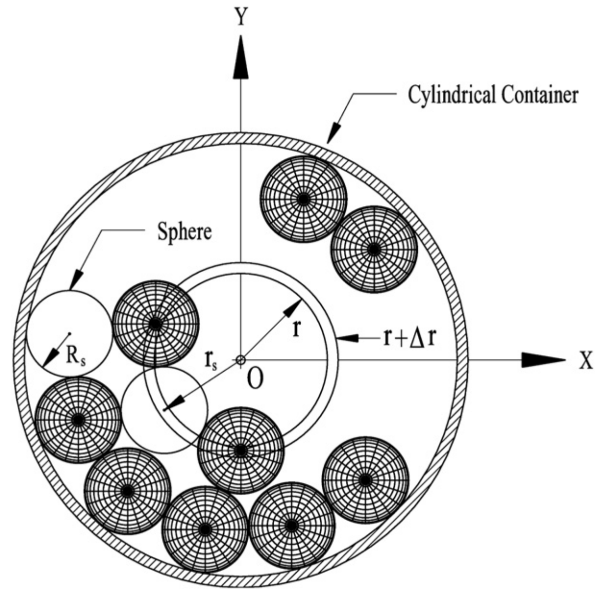

Fig. 1. X–Y plane of a radial annular layer at radial position r and $r+\Delta r$.

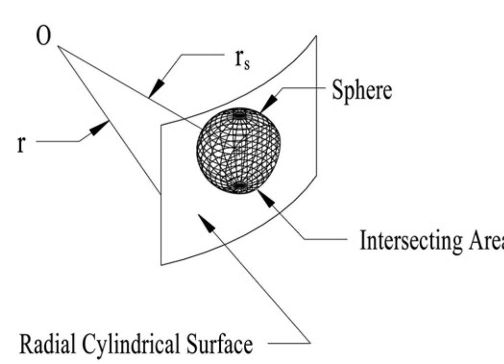

Fig. 2. Sphere intersecting area with radial cylindrical surface at radial position r.