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Discrete thermal element modelling of heat conduction in particle systems: Pipe-network model and transient analysis

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ABSTRACT

In our recent work [Y.T. Feng, K. Han, C.F. Li, D.R.J. Owen. Discrete thermal element modelling of heat conduction in particle systems: basic formulations. Journal of Computational Physics. 227: 5072–5089, 2008], a novel numerical methodology, termed the discrete thermal element method (DTEM), is proposed for the modelling of heat conduction in systems involving a large number of circular particles in 2D cases. The method cannot be easily extended to transient analysis, which causes difficulties in combining the DTEM with the conventional discrete element method for modelling thermal/mechanical coupling problems in particle systems. This paper presents a simplified version of the DTEM, termed the pipe-network model, in which each particle is replaced by a simple thermal pipe-network connecting the particle centre with each contact zone associated with the particle. The model essentially neglects the direct heat transfer between the contact zones and thus significantly simplifies the solution procedure of the original DTEM. With this feature, transient heat conduction analysis can now be performed in a straightforward manner. In addition, the entire algorithmic structure of the pipe-network model is compatible with the discrete element method, leading to an effective scheme for simulating thermal–mechanical coupling problems. Numerical experiments are conducted to establish the solution accuracy of the proposed model.

1. Introduction

In our recent work [1], a novel numerical methodology, termed the discrete thermal element method (DTEM), is proposed for the modelling of heat conduction in systems involving a large number of circular particles in 2D cases. In this framework, each particle is treated as an individual element with the average temperatures at the zones of contact with its neighbors as primary independent variables. The corresponding element thermal conductivity matrix can be very effectively evaluated and is entirely dependent on the characteristics of the contact zones, including the contact positions and contact angles. The element thermal stiffness matrix of the element shares the same form and properties as its conventional finite element counterpart. In particular, the entire solution procedure can follow exactly the same steps as those involved in the finite element analysis. Once the average temperatures at the contact zones are obtained, the temperature distribution field over each particle can be determined, which is a distinct advantage over the existing isothermal models (see for instance [4–7], and references therein). Thus this element offers a simple, effective and accurate computational procedure to simulate heat conduction in large scale particle systems encountered in many engineering and scientific applications.

The above work is part of a broader project which aims at developing coupled computational techniques [9–12] to effectively account for the interactions between different physical phases. A particle system with the simultaneous presence of thermal, mechanical and fluid phases is such a challenging problem, and is often characterised by a dynamic and transient nature. In this context, coupling the DTEM with the conventional discrete element method [8] would be, in principle, a possible option for modelling thermal/mechanical coupling problems in particle systems. Although the DTEM is capable of modelling the steady-state heat conduction in large particle systems very efficiently, the extension of the formulations to transient situations is not trivial. There are a number of ways to make the extension possible, but these are largely based on 'ad-hoc' and approximate procedures and thus may lose the mathematical rigour offered by the original discrete thermal element formulations.

The main objective of the current work is to propose a simplified discrete thermal element model which can be easily extended to transient analysis and is compatible with the conventional discrete element so that the thermal/mechanical coupling problem in particle systems can be tackled. Note that most of the existing isothermal models [2–7] are able to undertake transient analysis of heat conduction in particle systems. The main difference is that the present work provides a more rational and accurate model to represent heat conduction in circular particles.

The paper is organised as follows. First the basic discrete thermal element formulations are reviewed in the next section. Then a simplified

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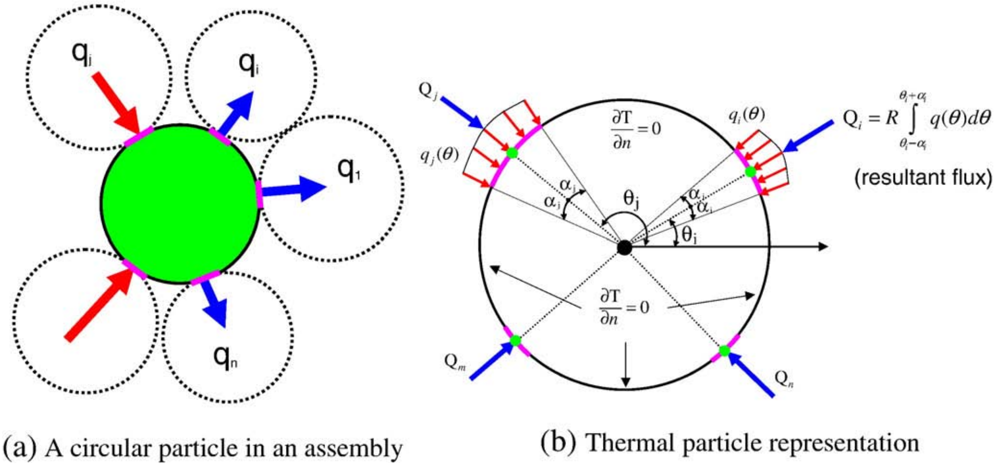

Fig. 1. Heat conduction in a simple particle system.

model, termed the pipe-network, will be proposed in Section 3, followed by the transient thermal analysis. Numerical experiments are conducted in Section 5 to assess the performance and solution accuracy of the proposed new methodology.

2. Discrete thermal element formulations

The basic formulations of the discrete thermal element method are summarised in the following. See [1] for more details.

2.1. Integral formulation

Consider a 2D circular particle of radius R in a particle assembly that is in contact with n neighboring particles, as shown in Fig. 1a, in which heat is conducted only through the n contact zones on the boundary of the particle, and the remainder of the particle boundary is fully insulated. A polar coordinate system (r, θ) is established with the origin set at the centre of the particle. Each contact zone, which is assumed to be an arc, can be described by the position of its middle point in terms of an angle θ and a contact angle α that determines the contact arc length, as shown in Fig. 1b. Generally the position angles θ_i of the contact zones are well spaced along the boundary and the contact angles α_i are small. The positions and contact angles, θ_i and α_i, of all the n contact zones constitute the local element (contact) configuration of the particle. Furthermore, if the heat flux along the i-th contact zone is described by a (local) continuous function $q_i(\theta)$, then the heat flux on the whole boundary of the particle can be represented as

$$q(\theta) = \begin{cases} q_i(\theta - \theta_i) & \theta_i - \alpha_i \le \theta \le \theta_i + \alpha_i \quad (i = 1, \dots n) \\ 0 & \text{otherwise} \end{cases}. \tag{1}$$

The heat flux equilibrium in the particle requires

$$\int_0^{2\pi} q(\theta) d\theta = 0. \tag{2}$$

The temperature distribution $T(r, \theta)$ within the particle domain $\Omega = \{(r, \theta): 0 \le r \le R;\ 0 \le \theta \le 2\pi\}$ is governed by the Laplace equation as:

$$\begin{cases} \kappa \Delta T = 0 & \text{in } \Omega \\ \kappa \dfrac{\partial T}{\partial n} = q(\theta) & \text{on } \partial\Omega \end{cases} \tag{3}$$

where κ is the thermal conductivity; $\partial\Omega$ denotes the boundary (circumference) of the particle; and $\frac{\partial T}{\partial n}$ is the temperature gradient along the normal direction to the boundary. Note that the flux $q(\theta)$ inward to the boundary is assumed positive here. Then the temperature at any point $(r, \theta) \in \Omega$ can be expressed as [13,1]

$$T(r,\theta) = -\frac{R}{2\pi\kappa}\int_0^{2\pi} q(\phi)\ln\left[1 - 2\frac{r}{R}\cos(\theta - \phi) + \left(\frac{r}{R}\right)^2\right]\mathrm{d}\phi + T_\mathrm{o} \quad (r,\theta)\in\Omega \tag{4}$$

where T_o is the temperature at the centre, i.e. $T_\mathrm{o} = T(0,0)$. In particular, the temperature on the boundary is obtained as [1]

$$T_\mathrm{c}(\theta) = T(R,\theta) = -\frac{R}{\pi\kappa}\int_0^{2\pi} q(\phi)\ln\left|\sin\frac{\theta - \phi}{2}\right|\mathrm{d}\phi + T_\mathrm{o} \tag{5}$$

or

$$T_\mathrm{c}(\theta) = -\frac{R}{\pi\kappa}\sum_{j=1}^{n}\int_{-\alpha_j}^{\alpha_j} q_j(\phi)\ln\left|\sin\frac{\theta - \phi - \theta_j}{2}\right|\mathrm{d}\phi + T_\mathrm{o}. \tag{6}$$

It is highlighted that the temperature at the centre T_0 is the average temperature not only along the boundary but also over the whole domain:

$$T_\mathrm{o} = \frac{1}{2\pi}\int_0^{2\pi} T_\mathrm{c}(\theta)\mathrm{d}\theta = \frac{1}{\pi R^2}\int_\Omega T(r,\theta)\mathrm{d}\Omega. \tag{7}$$

The solution (4), (5) or (6) is in integral form which provides an *explicit* formulation to evaluate the temperature distribution over the particle when the input heat flux along the boundary is given.

2.2. Discrete thermal element

From Eq. (6), the temperature distribution along the i-th contact arc is given by

$$T_\mathrm{c}^i(\theta) = -\frac{R}{\pi\kappa}\sum_{j=1}^{n}\int_{-\alpha_j}^{\alpha_j} q_j(\phi)\ln\left|\sin\frac{\theta - \phi - \theta_j}{2}\right|\mathrm{d}\phi + T_\mathrm{o} \quad (\theta_i - \alpha_i \le \theta \le \theta_i + \alpha_i). \tag{8}$$

Define T_i and Q_i respectively as the average temperature and the resultant flux on the i-th arc and further assume that $q_i(\theta)$ is *constant*. Then T_i can be obtained as

$$T_i = \sum_{j=1}^{n}\left[-\frac{Q_j}{4\pi\kappa\alpha_i\alpha_j}\int_{-\alpha_i}^{\alpha_i}\int_{-\alpha_j}^{\alpha_j}\ln\left|\sin\frac{\Delta\theta_{ij} + \theta - \phi}{2}\right|\mathrm{d}\phi\mathrm{d}\theta\right] + T_\mathrm{o} \tag{9}$$