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Translation of two rigid spheres perpendicular to their line-of-centers and normal to a plate

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ABSTRACT

The translation of two identical rigid spheres perpendicular to their line-of-centers normal to a rigid plate is analyzed theoretically for Reynolds number ranges from 0.1 to 40. The geometry considered allows us to examine simultaneously the effect of the presence of a boundary and that of nearby particles on the translation of a particle. We show that the presence of the plate has a significant influence on the flow field near the spheres, especially when Reynolds number is low. Due to the competition between the nozzle effect and the sphere–sphere interaction, the degree of the boundary effect on the drag coefficients of the spheres has a local minimum as the separation distance between two particles varies. In addition, the deviation of the ln (drag coefficient)–ln(Reynolds number) curve from a Stokes'-law-like relation may also have a local minimum as the separation distance between two spheres varies. An empirical relation is proposed to correlate the drag coefficient with the key parameters of the present problem for the case where Reynolds number is smaller than unity.

1. Introduction

The translation of particles in a fluid medium finds various applications in operations of both laboratory and industrial scales. Depending upon the operating conditions, the behavior of a translating particle can be influenced by its physical properties such as its shape, size, and relative density, the nature of fluid medium, and that of the applied field such as gravitational acceleration. In addition, the concentration of particles can be significant because the interaction of a particle with neighboring particles will certainly influence its behavior. Although the concentration effect is usually neglected in a laboratory study to avoid solving a complicated many-body problem, it needs be taken into account in practice because the concentration of particles is usually appreciable. In general, both the flow field surrounding an interactive particle and the drag acting on the particle can be different appreciably from those of an isolated particle. The settling speed of rigid particles in a dispersion, for instance, is slower than that of the corresponding isolated particle [1]. The analysis of the hydrodynamic interactions between two particles was originated by Stimson and Jeffery [2] through considering the translation of two identical rigid, coaxial spheres moving slowly along the line of their centers in an unbounded viscous fluid. Happel and Pfeffer [3] investigated experimentally the slow falling of two identical, rigid, coaxial spheres along the line of their centers in a viscous liquid. A micro-force system was developed by Zhu et al. [4] for the measurement of the drag acting on two interacting particles at a medium large Reynolds number. Using this technique, Liang et al. [5], Chen and Lu [6], and Chen and Wu [7] measured the drag acting on two interacting rigid spheres in a Newtonian fluid. Daugan et al. [8,9] investigated experimentally the settling of two or three identical particles along their line-of-centers in a shear-thinning fluid at a low Reynolds number. Hsu et al. [10] evaluated the drag acting on two coaxial, nonuniformly structured flocs in a uniform flow field. The translation of two nonuniformly structured flocs along the axis of a cylindrical tube was analyzed by Hsu et al. [11]. Hsu and Yeh [12] studied the translation of two coaxial rigid spheres along the axis of a cylindrical pore filled with a shear-thinning Carreau fluid.

Among the possible arrangements of particles, the case of two spheres translating side by side has been considered by many investigators. For example, Kim et al. [13] considered the case of a uniform flow past two spheres held fixed side by side for Reynolds number up to 150. Tsuji et al. [14] visualized the flow of two or three identical rigid spheres moving in an unbounded viscous fluid with Reynolds numbers smaller than 10^3, and measured the force acting on a sphere. They found that the drag acting on a sphere decreases with decreasing sphere–sphere distance. The same problem was solved numerically by Folkersma et al. [15] through a finite element method for small to medium large Reynolds numbers. Legendre et al. [16] calculated the drag and the lift forces acting on two identical spherical bubbles moving side by side in a viscous fluid for Reynolds number ranging from 0.02 to 500. Tsuji et al. [17] investigated an unsteady uniform fluid flow past two identical particles; both the case where the flow is

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parallel to the line connecting the centers of the spheres and that perpendicular to the line connecting the centers were considered. Schouveiler et al. [18] studied both experimentally and theoretically the problem of a uniform flow past two stationary spheres at low Reynolds numbers; the flow was perpendicular to the line connecting the centers of the spheres.

The presence of a boundary can also influence the behavior of a translating particle. In practice, this occurs, for example, in the sedimentation of particles in a relatively small vessel and/or if particles are considerably close to the vessel wall. In general, the presence of a boundary has the effect of raising the drag acting on a particle [19,20].

In this study, we consider the translation of two identical, rigid spheres perpendicular to their line-of-centers and normal to a rigid plate in a Newtonian fluid under the conditions of low to medium large Reynolds number. The geometry under consideration allows us to examine simultaneously both the presence of a boundary and the neighboring particles on the behavior of a target particle. In particular, the influences of the separation distance between two spheres, the distance between spheres and plate, and the Reynolds number, on the drag acting on the spheres are investigated in detail.

2. Mathematical modeling

Referring to Fig. 1, we consider the translation of two identical rigid spheres in an incompressible Newtonian fluid perpendicular to their line-of-centers toward a large rigid plate with velocity V. Let h, r_p, and S be the distance between the center of a sphere and the plate, the radius of a sphere, and the center-to-center distance between two spheres. The present problem is of two-dimensional nature, and the Cartesian coordinates (x, y) shown in Fig. 1 are adopted. For convenience, the spheres are remained fixed and the bulk liquid and the plate move with relative velocity V. Therefore, the governing equations and the associated boundary conditions for the flow field can be expressed as following [21]:

$$\rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla P + \mu \nabla \mathbf{u} \quad (1)$$

$$\nabla \cdot \mathbf{u} = 0 \quad (2)$$

$$u_x = V \text{ at } x = 0 \quad (3)$$

$$u_x = V \text{ as } y \rightarrow \pm\infty \quad (4)$$

$$u_x = 0 \text{ on the sphere surface} \quad (5)$$

Here, ∇ is the gradient operator, P is the pressure, and ρ, $\boldsymbol{\mu}$, and $\mathbf{u}$, are the density, the viscosity, and the velocity of the fluid, respectively, and $\mathbf{u}_x$ is the x-component of $\mathbf{u}$.

For the translation of an isolated rigid sphere in an infinite Newtonian fluid in the creeping flow regime the drag coefficient C_D and Reynolds number Re are related by the Stokes law [21–23], $C_D = 24/Re$. In the present case, that relation needs be modified as

$$C_D = \frac{A}{Re}, \quad (6)$$

where A is a function of S, h, r_p, and Re. C_D can be evaluated by [21,22]

$$F = \left(\frac{1}{2}\rho V^2\right)\left(\pi r_p^2\right) C_D, \quad (7)$$

where F is the hydrodynamic drag acting on the spheres, which is obtained by first solving Eqs. (1) and (2) subject to Eqs. (3)–(5), and then evaluate the relevant forces acting on the spheres. FIDAP 7.0 [24] is adopted to solve the governing equations and the associated boundary conditions. To test its applicability, the translation of two rigid spheres perpendicular to their line-of-centers considered by Happel and Brenner [19], where analytical result is available for creeping flows, is solved by that software. The results obtained are summarized in Table 1. For creeping flows, the drag on the left sphere should be the same as that on the right sphere. The small difference between the numerically calculated drags on the spheres seen in Table 1 arises mainly from Re is not small enough and the precision limit of the software. The level of the deviation, however, suggests that the performance of the software adopted is satisfactory.

Table 1
Drag on two side-by-side rigid spheres for various values of (S/r_p) at $Re = 0.1$; analytic results are derived by Happel and Brenner [19], numerical results are those obtained in this study.

S/r_p	Analytic (N)	Numerical, left sphere (N)	Percentage deviation (%)	Numerical, right sphere (N)	Percentage deviation (%)
3	7.32497E−05	7.47071E−05	1.98954	7.53580E−05	2.87821
4	7.90354E−05	7.87338E−05	−0.38149	7.89332E−05	−0.12923
5	8.13411E−05	8.16999E−05	0.44117	8.05113E−05	−1.02013
6	8.36468E−05	8.48479E−05	1.43600	8.45428E−05	1.07241
10	8.76385E−05	8.75717E−05	−0.07620	8.77960E−05	0.17979
14	8.94425E−05	8.89436E−05	−0.55780	8.86008E−05	−0.94106
20	9.0837E−05	9.08068E−05	−0.03319	9.08668E−05	0.03283

3. Results and discussion

The influences of the parameters key to the present problem, including (S/r_p), (h/r_p), and Re, on the behaviors of the flow field and C_D are investigated through numerical simulation. Double precision is used throughout the computation, and grid independence is checked. In the latter, using roughly 9×10^6 elements in the liquid domain is sufficient for the ranges of the parameters considered.

Some typical flow fields simulated at various combinations of (h/r_p), (S/r_p), and Re are presented in Figs. 2 and 3. As seen in Fig. 2, boundary layer separation occurs in the rear region of a sphere at $Re = 40$ and the

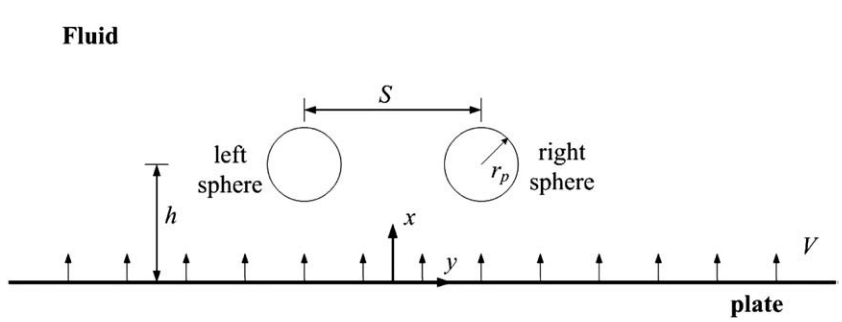

Fig. 1. The translation of two identical rigid spheres of radius r_p perpendicular to their line-of-centers normal to a large rigid plate; h is the distance between the center of a sphere and the plate, S is the center-to-center distance between two spheres, and (x, y) is the Cartesian coordinates adopted. The spheres are fixed in space and the bulk fluid and the plate move with a relative velocity V.