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A modified kinematic model for particle flow in moving beds

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Abstract

Granular materials are utilized in a wide variety of industrial applications. The complex hydrodynamics of granular flow is still not fully understood. The flow characteristics of solid particles in a two-dimensional moving bed were studied experimentally and theoretically. The effects of mass velocity, properties of particles and the shape of moving bed on flow pattern were examined. The experiments show that the mass velocity of solid particles and the placement of baffles in stagnant zones have no effect on flow pattern. The shape, size and repose angle of particles affect the flow pattern greatly. The particles have more liquidity and the mixture between particles is easier with the increasing of sphericity degree of particles and the decreasing of particle size. The shape of flowing zone becomes more sharp-angled with the increasing of the repose angle, and the stagnant zone becomes larger. The kinematic model was used to simulate the particle flow. For center discharge the simulation results are in agreement with experiment data very well, while for eccentric discharge the simulation couldn't show the effects of side walls on particle flow. An analytic solution of kinematic model was obtained by Lie transformation group. It shows that the velocity of particle flow is the maximum when x equals 0. The method of translational coordinate was used to modify the traditional kinematic model in order to make sure the x value of each point on the maximum particles velocity distribution lines is zero. The modified kinematic model could simulate the eccentric discharge well, and the field of kinematic model applications was expanded.

Keywords: Moving bed; Particle flow; Kinematic model; Modified model; Simulation

1. Introduction

Granular materials are widely used in the chemical industry with particulate reaction engineering, design of powders, storage of grain flows, separation and granulation. Roughly 1/2 of the products and about 3/4 of the raw materials of chemical industry are in form of granular materials [1]. Granular materials display a surprisingly complex range of properties, which make them appear solid- or liquid-like depending on the applied conditions [2]. The expertise gained in general modeling and prediction of the physical behavior in the granular flow is universally recognized as still incomplete [3].

In the past, the major theoretical methods were based upon plasticity theory for the analysis of hopper flow in the slow flow regime. Tüzün [4] reviewed the work of plasticity theory in the fields. Litwinszyn [5] first considered the possibility that the velocities are determined by purely kinematic effects, and presented the kinematic model for granular slow flow under gravity. Later, Mullins [6–8] independently proposed an equivalent stochastic model of the flow in terms of "void" and extensively developed the continuum limit. Subsequently, Nedderman and Tüzün [9] have produced a third variant on this approach from a constitutive law relating horizontal velocity and vertical velocity gradient. Shvab [10] derived the kinematic model from the same idea as Nedderman by the method of dimensionalities, and get different model equations. In spite of some basic differences in the formulation of their theories, the flow models all recognize the discrete nature of the particulate medium in contrast with the stress-based approach that considers the material as a plastic continuum.

In light of its simplicity, many experiments on silo drainage were viewed as successes of the model, and it has been accepted by engineering. Nedderman [11] presented a modified kinematic model and used it to predict the changing shape of the flowing zone and the development of the stagnant zone boundary in the batch discharge of bunker. He found that the zone boundary moves up the streamline with a velocity of $\rho_2/(\rho_1-\rho_2)$ times

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the velocity with which the particles at the same point move (where, ρ_1 and ρ_2 are the granular packing density in the stagnant zone and flowing zone, respectively). Zhang [12] used kinematic model predict the flow pattern of iron ore pellets in a full-scale silo and polypropylene pellets in a half cylindrical model silo. The predicted resident times of the markers were found to be in good agreement with the measurement values.

Choi [2] considered that the linear relationship between horizontal velocity and gradient of vertical velocity was too simple to capture all aspects of the flow profile. They inferred that the kinematic constant B increases with the local velocity. Hsiau [13] employed kinematic model to evaluate the flow in the exit of symmetric louver section. They found that the absolute value of the kinematic coefficient B is increased with the increase of the louver angle.

The kinematic model is mostly used in center discharge process. Chou [14,15] employed the kinematic model in a two-dimensional flat-bottomed hopper with eccentric discharge. They postulated that the kinematic constant B was proportional to the height of the stagnant zone, and presented the vertical particle velocity component on both sides of the outlet as series equations by separation of variables. When the outlet is appressed to the side wall, they cannot describe the granular flow accurately either.

In this research, with the employment of the Nedderman and Tüzün kinematic model for granular media in a two-dimensional flat-bottomed hopper with eccentric discharge, a boundary-value problem was constructed. The flow of granular in the moving bed was measured and compared with the results of kinematic model. An analytic solution of kinematic model was obtained by Lie transformation group. A modified kinematic model was presented for eccentric discharge, and the field of the kinematic model application to granular flows was expanded.

2. Analytic solution of kinematic model

2.1. Kinematic model

Nedderman and Tüzün [9] considered three particles as shown in Fig. 1. If the two particles in the lower layer have different velocities there will be a tendency for the upper particle to move sideways towards the faster falling particle. This can be expressed algebraically as:

$$u = f\left(\frac{\partial v}{\partial x}\right) \tag{1}$$

and it was noted that the simplest form of this relationship:

$$u = -B\frac{\partial v}{\partial x} \tag{2}$$

Assuming the density fluctuation was small in dense granular regimes, then combine Eq. (2) with the incompressibility condition

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

and obtained the equation for the kinematic model:

$$\frac{\partial v}{\partial y} = B\frac{\partial^2 v}{\partial x^2} \tag{4}$$

where B is the kinematic constant. B has the dimensions of length and is proportional to length scale of particle size. The experimental values of B by Nedderman [9] are two to three times the particle diameter. In this research, the value of B is set 2.5 times the particle diameter.

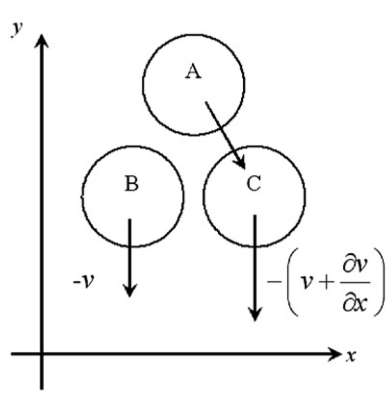

Fig. 1. The kinematic model of Nedderman and Tüzün.

2.2. Analytic solution of kinematic model by Lie transformation group

Lie transformation group method is one of the most effective ways for solving nonlinear partial differential equations (PDE). Here, we assume that there exists a solution of Eq. (4) and along a family of curves, called similarity curves, for which the equation of PDE reduces to a set of ordinary differential equations (ODE); this type of solution is called a similarity solution [16]. In order to determine a similarity solution we seek a one-parameter group (OPG) of transformations:

$$\begin{cases} v_1 = v + \varepsilon\eta(x,y,v) + O(\varepsilon^2) \\ x_1 = x + \varepsilon\xi(x,y,v) + O(\varepsilon^2) \\ y_1 = y + \varepsilon\tau(x,y,v) + O(\varepsilon^2) \end{cases} \tag{5}$$

where $\eta(x,y,v)$, $\xi(x,y,v)$, $\tau(x,y,v)$ are the infinitesimality of x, y, v respectively, and they are to be determined in such a way that the PDE (4) is constantly conformally invariant with respect to the transformation Eq. (5):

$$\frac{\partial v_1}{\partial y_1} = B\frac{\partial^2 v_1}{\partial x_1^2} \tag{6}$$

ε is an infinitesimality and chosen so small that its square and higher order terms may be neglected. The existence of such a group allows the number of independent variables to be reduced by one, thereby allowing the PDE (4) to be replaced by a system of ODE.

Here, we use transformation Eq. (5) to employ constantly conformally invariance of the PDE (4). Substituting the Eq. (5)