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Short communication

Addressing an inverse problem of classifier size distributions

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Abstract

One input and two output stream classifiers are commercially employed for the classification of particles. A mass balance equation for a classifier suggests that the feed size distribution can be evaluated from measured product size distributions if and only if the flow split of the feed particles to one of the product streams is also known. Moreover, the mass balance equation used to reconcile measured size distributions indicates that flow split of solid particles is in turn a function of all the three size distributions and is then redundantly expressed over the mass fraction of particles retained in various discrete size classes. Therefore for an operating classifier under steady state, the so far recognized approaches fail to address the profile of feed size distribution from the knowledge of measured fine and coarse product size distributions alone. In the forward approach of estimation of product size distributions, the feed distribution is integrated with efficiency curve of the classifier. Thus as an inverse problem, the feed distribution and efficiency curve need to be identified from the measured product size distributions. This paper attempts to address this inverse problem when flow split of feed particles to product streams is not known. However the method considers additional information regarding the functional forms of the classifier distributions due to inadequacy of product distributions alone to address the inverse problem.

Keywords: Classifier; Inverse problem; Particle size distributions

1. Introduction

A variety of size classifiers such as screens, cyclones, mechanical screw and rake classifiers and hydraulic classifiers are industrially employed for classification of particles. The size distribution of classifier is a crucial information that depicts the performance of classifier and influences the downstream operations or affects the product quality wherein it is used. Plant audits and reconciliation of the measured size distributions help in the assessment of the plant performance as well as suggest possibilities for improvement of the performance of the circuit and hence are essential to be carried out in process industries on a regular basis [1–4]. Many a times the design of the plant does not permit the sampling of all the streams during plant audit and a compromise on the data collection points has to be made depending on the availability of sampling point for each stream. The missed out information for various streams of the circuit during a plant audit is rebuilt when permissible by way of reconciliation of the measured data subjected to mass balance constraints [5].

Classification of particles is generally studied with regard to the attributes like size and/or density, and classification results into two or more product streams. In case of multiple product streams, classification can be viewed as a network of one input and two output separations occurring in series [6,7]. A mass balance of one input and two output stream separator is written as

$$F(d) = S_{\text{split}}U(d) + (1-S_{\text{split}})O(d) \tag{1}$$

where S_{split} represents solid flow split of feed particles to coarse stream, which is also called total-efficiency of separation and $F(d)$, $U(d)$ and $O(d)$ respectively represent cumulative percent finer size distributions of feed, coarse and fine streams.

For discrete size distributions obtained from sieve analyses, Eq. (1) can be written as

$$f(d_i) = S_{\text{split}}u(d_i) + (1-S_{\text{split}})o(d_i) \tag{2}$$

where d_i denotes a representative size of the i-th size class of particles and $f(d_i)$, $u(d_i)$ and $o(d_i)$ respectively represent the

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mass fraction of particles retained in the i-th size class of feed, coarse and fine streams. Rewriting Eq. (2) in the following form expresses S_{split} redundantly over the size classes and this information is used in reconciliation of error prone measured data so as to make the distributions consistent with Eq. (2) [7].

$$S_{\text{split}} = \frac{f(d_i) - o(d_i)}{u(d_i) - o(d_i)} \tag{3}$$

The actual separation efficiency also referred as grade efficiency of separation [8] can be calculated from the measured discrete mass fractions from the following expression. These efficiency factors are called partition coefficients for specified sizes.

$$E_a(d_i) = S_{\text{split}} \frac{u(d_i)}{f(d_i)} \tag{4}$$

Eqs. (1) and (2) suggest that feed size distribution can be obtained from the product size distributions only when the solid flow split of feed particles to one of the product streams, say to the coarse stream, S_{split}, is known. It appears from Eqs. (1) and (2) that there are infinitely many feed distributions that can produce, same coarse and fine distributions depending on the value of S_{split}. Therefore based on the chosen value of S_{split} to evaluate feed distribution, there exist a corresponding efficiency curve that varies between by-pass fraction of the classifier and 1, which when coupled with feed distribution gives the two product size distributions (refer Eqs. (7) and (8)). For example, Fig. 1 shows that the two product size distributions namely fine and coarse distributions can be obtained by respectively combining feed distributions (i), (ii) and (iii) in Fig. 1(a) with efficiency curves (i), (ii) and (iii) in Fig. 1(b). This non-unique solution makes this problem an ill posed inverse problem and the question is to identify the proper feed distribution and efficiency curve that gave the measured product size distributions for an operating classifier in the absence of measured S_{split} value. Since the measured data of product distributions are alone insufficient to address this problem, an additional hypothesis regarding the functional forms of the product distributions that are obtained by combining restricted functional forms of feed and actual efficiency curve are invoked.

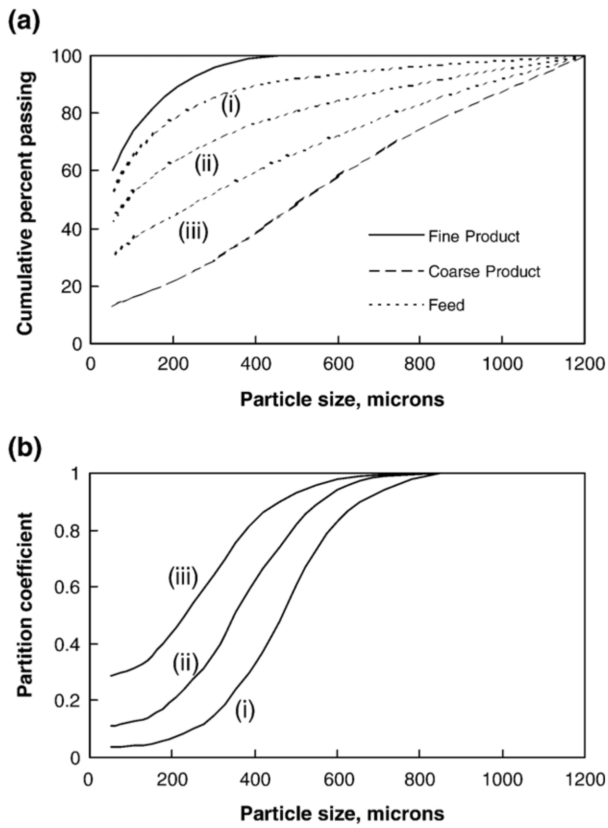

Fig. 1. Characterization of inverse problem: Product size distributions in (a) can be obtained by combinations of feed distributions (i), (ii) and (iii) in (a) with corresponding efficiency curves (i), (ii) and (iii) in (b).

2. Mathematical treatment

Plitt [9] proposed an actual efficiency curve of the following form that gives probability of feed of given size, d, reporting to the coarse stream.

$$E_a(d) = (1 - R_f)E_c(d) + R_f \tag{5}$$

where the corrected efficiency curve, $E_c(d)$, is given by

$$E_c(d) = 1 - \exp\left(-\ln(2)\left(\frac{d}{d_{50c}}\right)^m\right) \tag{6}$$

Recently, truncated analytical expressions for cumulative percent passing distributions for the finer and coarser streams have been derived from the knowledge of feed and efficiency curve [10] by considering

$$O(d) = 100 \frac{\int_0^d (1 - E_a(d))f(d)\,\mathrm{d}d}{\int_0^{d_{\max}} (1 - E_a(d))f(d)\,\mathrm{d}d} \tag{7}$$

and

$$U(d) = 100 \frac{\int_0^d E_a(d)f(d)\,\mathrm{d}d}{\int_0^{d_{\max}} E_a(d)f(d)\,\mathrm{d}d} \tag{8}$$

Under restricted functional forms of feed distribution represented in terms of Gates–Gaudin–Schumann (GGS) function and classifier efficiency represented in terms of Plitt function [10], the cumulative form of feed, fine and coarse stream distributions of the classifier are respectively represented by

$$F(d) = \begin{cases} 100\left(\frac{d}{d_{\max}}\right)^n & \text{for } 0 \le d \le d_{\max} \\ 100 & \text{for } d > d_{\max} \end{cases} \tag{9}$$