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Settling velocity of cubes in Newtonian and power law liquids

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Abstract

New extensive data on the free settling velocity of thirty cubes of various densities and sizes falling in scores of Newtonian and Power law liquids are reported herein to supplement the existing data, for there is very little prior data on cubes in power law liquids. The new data embrace the range of conditions as follows: sphericity of 0.805; power law index, 0.61 to 1 and consistency index, 0.0078–15.31 Pa s^n; Reynolds number, 0.0013 to 860. The new results are shown to be consistent with an existing drag correlation which has been tested extensively using the literature data for spherical and non-spherical particles falling in Newtonian and power law liquids with acceptable levels of accuracy.

Keywords: Cubes; Settling velocity; Drag coefficient; Power law; Sphericity

1. Introduction

Owing to the frequent occurrence of liquid–solid systems in a wide range of industrial settings, considerable research efforts has been devoted to the development of reliable expressions to predict the free settling velocity of spherical and non-spherical particles in Newtonian [1–5] and in power law type non-Newtonian liquids [6–11]. It is customary to express this information using the standard dimensionless parameters, namely, drag coefficient, Reynolds number and sphericity in Newtonian media, with the further inclusion of the power law index for settling in power law liquids. Since most of the pertinent studies have been thoroughly reviewed recently [12,13], only their salient features are recapitulated here. For spherical particles, based on a combination of numerical and experimental results, satisfactory schemes are now available for a priori estimation of the settling velocity of a sphere in Newtonian media of known physical characteristics [12–14] under most conditions of interest. In contrast, much less information is available on spheres falling in power law fluids [6,9,13,15,16]. Reliable numerical predictions of sphere drag in power law liquids are now available up to the Reynolds number values of 500 and of the power law index ~0.5 to 1.8 [16]. These ranges are supplemented by experimental results up to the Reynolds number values of ~1500 [13,16,17]. Owing to their generally high viscosity levels, in most applications, settling of particles in power law liquids usually occurs at low to moderate Reynolds numbers and therefore the upper limit of $Re=1500$ of currently available results is not as serious as it might appear.

In contrast, even less is known about the settling velocity of non-spherical particles both in Newtonian and in power law fluids. Extensive evaluations of the available data against the predictive correlations have been reported amongst others by Chhabra et al. [1], Tang et al. [3] and Yow et al. [4] for regular shaped particles falling in Newtonian media and by Rajitha et al. [8] in power law liquids whereas the prediction of drag of irregular shaped particles in Newtonian fluids has been dealt with by Tran-Cong et al. [2]. During the course of these extensive evaluations, it became clear that very few experimental results are available for cubes falling freely even in Newtonian liquids, let alone in power law liquids. For instance, particles of this shape are frequently encountered in food processing applications, e.g. carrot slices are of almost cubical shape. Aside from such practical applications, there is an intrinsic interest in studying cubes as these constitute regular shaped particles with a constant value of sphericity and thus allow the delineation of the effects arising from the shear-

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thinning characteristics of the liquids and from the non-spherical shape of the particles. Hence, such studies contribute to the overall understanding of the settling behavior of non-spherical particles in non-Newtonian fluids. This study aims to fill this gap in the current body of information.

New extensive experimental results are reported in this work on the settling velocity of thirty cubes made of different size and of six different materials falling in several Newtonian and power law liquids, thereby encompassing wide ranges of the particle Reynolds number and of the power law index. Detailed analysis revealed that these results are consistent with the predictions of an existing expression which has already been tested extensively for non-spherical particles settling in Newtonian (using 1900 individual data points) and power law liquids using 1437 individual data points [6].

2. Experimental

Thirty cubes made of brass (8403 kg/m^3), of stainless steel (7741 kg/m^3), of aluminium (2684 kg/m^3), of teflon (2123 kg/m^3), of acrylic (1173 kg/m^3) and of nylon (1137 kg/m^3) ranging in size from 6 mm to ~25 mm have been used in this study thereby covering wide ranges of particle size and density. The sphericity of cubes is, of course, fixed at $\psi = 0.805$. In addition, a few spheres of teflon and steel were also used to ascertain the reliability and accuracy of the experimental protocols thereby establishing the extent of uncertainty for the results reported in this work.

Eight glucose-in-water solutions of varying concentrations were used as the model Newtonian experimental test fluids. Five aqueous Carboxymethyl Cellulose (CMC of medium viscosity grade, marketed by Loba chemicals, Mumbai, India) solutions of varying concentrations (0.9–3.2% w/w) were employed as model power law fluids. The polymer solutions were prepared by slowly adding the required amount of dry polymer powder to water in an agitated tank. In order to minimize the extent of bacterial degradation, polymer solutions were seeded with a few ppm concentration of Formalin.

The density of each test fluid was measured using a constant volume density bottle whereas the steady shear data of each solution was obtained using the concentric cylinder configuration in a Bohlin CVO-100 rheometer at the same temperature as that encountered in the settling experiments. The rheological characterization of CMC polymer solutions was carried out over the shear rate range of 0.1 to 200 s^{-1} and the corresponding estimates of the shear rate values produced by settling cubes, approximated as ($2V/d_s$), are well within this range. Furthermore, no measurable viscoelasticity (storage modulus in oscillatory shear) was observed in the highest concentration CMC solution used in this work and therefore the polymer solutions used here are believed to be inelastic shear-thinning liquids.

A clear glass tube (of 100 mm inside diameter and 1500 mm long) was used to perform the settling experiments. Each test liquid was loaded into the fall tube for a long period of time for the air bubbles to escape and for the thermal equilibrium to be reached before performing the particle sedimentation tests. Similarly, the test particles were also soaked in the test liquids prior to being released in the liquid in order to eliminate the possibility of any air entrainment. Test particles were dropped as close to the centre of the fall tube as possible and their descent was timed using an electronic stop watch reading up to 10 ms. In most cases, cubes retained their original orientation, except a few falling at high velocities in water. The descent of a particle was timed over two test sections, both of which were located sufficiently away from the two ends for the end effects to be negligible [13,14] and from each other to ascertain that the particle had attained its terminal falling velocity. Only tests with concordant fall times over the two test sections were finally accepted. Furthermore, each particle was dropped at least twice and the reproducibility of the results is of the order of 1.5%. The terminal settling velocity measured in this work ranged from ~0.76 mm/s to 1 m/s. Altogether 159 individual data points (70 in glucose solutions and 89 in CMC solutions) have been gleaned in this study to elucidate the dependence of the free settling velocity of cube-shaped particles on fluid properties.

3. Results and discussion

3.1. Physical properties of test liquids

As expected, aqueous glucose solutions exhibited constant shear viscosity thereby confirming their Newtonian flow behavior. Table 1 summarizes the values of their density and viscosity; the latter is seen to vary by a factor of ~1000. On the other hand, as expected, all polymer solutions used here displayed varying levels of shear-thinning behavior which could be adequately approximated by the usual two parameter power law model, i.e.;

$$\tau = m(\dot{\gamma})^n. \tag{1}$$

Using the steady shear data, the best values of the consistency index, m and of the flow behavior index, n were established using the non-linear regression. These values are also summarized in Table 1. While the values of m show a four fold variation, the test fluids show a moderate degree of shear-

Table 1
Properties of test liquids

(w/w %)	Fluid	T (K)	n (–)	m (Pa s^n)	ρ (kg/m^3)
19.5	Glucose	297	1	0.0078	1099
23	Glucose	298	1	0.0081	1113
23.8	Glucose	298	1	0.0092	1159
56	Glucose	305	1	0.417	1354
67	Glucose	306	1	2.15	1371
75.9	Glucose	296	1	4.04	1385
77	Glucose	295	1	4.10	1389
90.6	Glucose	307	1	8.46	1390
3.2	CMC	299	0.61	15.31	1018
2.5	CMC	299.4	0.79	8.27	1017
1.8	CMC	300.4	0.81	5.08	1007
1.2	CMC	300.2	0.79	4.25	1009
0.90	CMC	301	0.79	3.79	1016