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Drag coefficient and settling velocity for particles of cylindrical shape

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Abstract

Solid particles of cylindrical shape play a significant role in many separations processes. Explicit equations for the drag coefficient and the terminal velocity of free-falling cylindrical particles have been developed in this work. The developed equations are based on available experimental data for falling cylindrical particles in all flow regimes. The aspect ratio (i.e., length-over-diameter ratio) has been used to account for the particle shape. Comparisons with correlations proposed by other researchers using different parameters to account for the geometry are presented. Good agreement is found for small aspect ratios, and increasing differences appear when the aspect ratio increases. The aspect ratio of cylindrical particles satisfactorily accounts for the geometrical influence on fluid flow of settling particles.

Keywords: Drag coefficient; Drag force; Settling velocity; Terminal velocity; Cylindrical particles

1. Introduction

Many processes for the separation of particles of different sizes and shapes depend upon variations in the behavior of the particles when subjected to the action of a moving fluid. A particle falling in an infinite fluid under the influence of gravity will accelerate until the gravitational force is exactly balanced by the resistance force that includes buoyancy and drag. The constant velocity reached at that stage is called the "terminal velocity." The resistive drag force depends upon an experimentally determined drag coefficient.

Drag coefficients and terminal velocities are important design parameters in many separation processes. Many equations have been developed and presented in the literature relating the drag coefficient (C_D) to the Reynolds number (Re) for particles of spherical shape falling at their terminal velocities. These correlations are of varying complexity and contain many arbitrary constants. Many of these correlations are listed in Clift et al. [1], Khan and Richardson [2], and Haider [3].

In the case of nonspherical particles, less information is found in the literature. Heywood [4] developed an approximate method for calculating the terminal velocity of a nonspherical particle or for calculating its size from its terminal velocity. The method was an adaptation of his method for spheres. Heywood used an empirical factor (k) to account for deviations from the spherical shape.

Haider and Levenspiel [5] presented a generalized C_D-vs.-Re correlation for nonspherical particles. They used the concept of sphericity (ϕ), originally introduced by Wadell [6], to account for the particle shape. The authors also reported a correlation to calculate explicitly terminal velocities for particles of different shapes. However, cylinders and needles are usually non-isometric particles, therefore, other parameters maybe more appropriate to account for particle shape. There are also several ways to define the characteristic length to be used in the required dimensionless numbers. Clift et al. [1] summarized the different alternatives.

Predictions by Haider and Levenspiel [5] showed relatively poor accuracy for particles with $\phi<0.67$, therefore, some authors [7–10] attempted to improve the accuracy of the Haider and Levenspiel [5] correlations. Chien [7] and Hartman et al. [8] used the sphericity as shape factor. A somewhat different approach was presented by Thompson and Clark [9]. These authors defined a shape factor (ε), which is simply the ratio of the drag coefficient for the non-spherical particles to that of a sphere, both evaluated at $Re=1000$. The problem found in this approach rests on the prediction of the shape factor. Thompson and Clark [9]

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failed in their attempt to link the shape factor ε with the sphericity, the second harmonic, and the Corey shape factor. Ganser [10] used the fact that every particle experiences a Stokes regime where the drag is linearly related with the velocity and a Newton regime where the drag is proportional to the square of the velocity. Ganser [10] introduced two shape factors, K_1 and K_2, applicable in the Stokes and Newton regimes, respectively. The two shape factors were found to be unique functions of the sphericity [10]. The author also presented explicit correlations for C_D and Re number.

Chhabra et al. [11] collected experimental results of 19 independent studies comprising several different particle shapes, including cylinders. The resulting data base consisted of 1900 experimental points covering wide ranges of physical properties and kinematic conditions. The authors used the collected data to compare different available correlations published in the literature.

In the case of particles of cylindrical shape several authors have presented correlations of experimental data [12–18]. Clift et al. [1] reviewed several of these correlations.

The main goal of this work is to develop a generalized correlation for calculation of the terminal velocity of cylindrical particles for a wide range of geometric and flow conditions. The proposed correlation is compared with several general correlations available in literature. The choice of an appropriate shape factor for cylindrical particles is discussed.

2. Shape factors

Natural and man-made solid particles occur in almost any imaginable shape from roughly spherical pollen and fly ash to cylindrical asbestos fibers and irregular mineral particles. Axisymmetric particles are among the most commonly found. The group comprises bodies generated by rotating a closed curve around an axis. Spheroidal and cylindrical particles of various kinds are of particular interest because they correspond closely to the shapes adopted by many drops and bubbles and to the shapes of some solids. Axisymmetric particles are conveniently described by the aspect ratio (E), defined as the ratio of the length projected on the axis of symmetry to the maximum diameter normal to the axis [1].

Most particles of practical interest are irregular in shape. A variety of empirical factors have been proposed to describe nonspherical particles and correlate their flow behavior. Empirical description of the particle shape is provided by identifying two characteristic parameters from the following list [19]:

1. volume, V;
2. surface area, A;
3. projected area, A_p; and
4. projected perimeter, P_p.

The projected area and perimeter must be determined normal to some specified axis. For axisymmetric bodies, the reference direction is taken parallel or normal to the axis of symmetry. An "equivalent sphere" is defined as the sphere with the same value of one of the above parameters. The "particle shape factor" is defined as the ratio of a characteristic parameter from the above list to the corresponding value for the equivalent sphere [1].

Heywood [20] proposed a widely used empirical parameter based on the projected profile of a particle. The "volumetric shape factor" is defined as

$$k = V/d_A^3, \tag{1}$$

where $d_A = (4A_p/\pi)^{0.5}$ is the projected area diameter, which is calculated as the diameter of a sphere with equal projected area as that of the particle, and A_p is the projected area of the particle. The projected area of the particle is a difficult parameter to determine because it depends upon the orientation of the particle. A number of methods have been suggested to estimate d_A without knowing A_p [1].

Wadell [6] proposed that the degree of sphericity be defined as

$$\phi = A_V/A, \tag{2}$$

where A_V is the surface of a sphere having the same volume as the particle, and A is the actual surface area of the particle. According to this definition, the sphericity of a true sphere is equal to 1. The more the aspect ratio departs from unity, the lower is the sphericity. In the case of irregular particles, it is difficult to determine ϕ directly.

The sphericity (ϕ) of the particles was used by Haider and Levenspiel [5] to account for the particle shape of isometric particles, which are particles with similar sizes for all significant dimensions. The authors used experimental data from nonspherical shapes such as cubes, octahedrons, tetrahedrons, other nonspherical shapes, and free-falling thin disks. For isometrically shaped particles, the sphericity is considered the best single parameter for describing the shape of falling particles [5].

Wadell [21] also introduced the "degree of circularity" as

$$\psi = P_A/P_p = \pi d_A/P_p, \tag{3}$$

where P_A is the perimeter of a sphere with equivalent projected area, and P_p is the projected perimeter of the particle. Unlike the sphericity, ψ can be determined from microscopic or photographic observation. The use of ψ is only justified on empirical grounds, but it has the potential advantage for allowing the correlation of flow dependence on particle orientation.

In the case of axisymmetric particles with creeping flow parallel to the axis of symmetry, Bowen and Masliyah [19] found that the most useful shape parameter was

$$\Sigma = A/A_p. \tag{4}$$

Some authors [22,23] have used the so called Corey shape factor, β, defined as,

$$\beta = c/(ab)^{1/2}, \tag{5}$$

where $a > b > c$ are the three principal axes of the particle. The shortcomings of such shape factor have been discussed by Alger